Optimization Models for Industrial Process Targeting

by

Mohammad Farrukh Shair Pulak

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

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كلية الدراسات العليا

COLLEGE OF GRADUATE STUDIES

Dedicated to

the late memories of my Grand Fathers

MOHAMMAD KHORSHADE ALAM

&

MOULOVI MASER ALI MONDOL

Acknowledgments

Praise be to the Lord of the World, the Almighty for having guided me at every stage of my life.

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Special thanks to my wife Glory, who suffered for a long time to present me a son *Thabit Farrukh.* Glory's tolerance, love, understanding, encouragement and paticntly endurance from my deprivation and neglect during the preparation and revision of this thesis, made it possible to complete my M.S program.

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Abstract

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Title: Optimization Models For Industrial Process Targeting

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Recently, there has been a lot of interest in the economics of quality control. One of the important issues in quality control is finding the optimal target value for a process under various inspection policies. Researchers have considered various aspects of this problem either with 100% inspection or with quality sampling plans, but none of them has considered this problem when rectifying inspection is used. We propose developing various targeting models for one machine when rectifying inspection is used, and for two machines in series under various inspection policies. We also investigate the effect of machine variance on the optimum target value. Finally, we develop a computer package to solve various targeting models in the literature.

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ملخص الرسالة

الاسم	: محمد فاروق بولاك
العنوان	: نماذج مثلى لتحديد الهدف في العمليات الصناعية
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لقد ازداد في الآونة الأخيرة الإهتمام باقتصاديات ضبط الجـودة، ومـن اهم المواضيع في ضبط الجودة هو موضوع تحديد قيمة الهدف المثلى لعملية صناعية تحت سياسات فحص مختلفة. لقد نظر الباحثون في هذه المشكلة من عدة وجوه سواءً مع فحص ١٠٠٪ أو مع خطط أخذ العينات لفحص الجودة، ولكن لم يدرس احد من هؤلاء البـاحثين هذه المشـكلة مـع الفحـص المعـالج. نعرض في هذا البحث نماذج مثلى مختلفة لتحديد الهدف لكل من آلـة واحـدة مع فحص معالج ولآلتين متتاليتين مـع سياسـات فحـص مختلفـة. كمـا نبحـث تأثير تباين الآلة على تحديد قيمة الهدف المثلى. وأخيرا نطور حزمة برمجية للحاسب الآلي لحل النماذج المختلفة المنشورة سابقا لتحديد قيمة الهدف.

درجة ماجستير العلوم

جامعة الملك فهد للبترول والمعادن

الظهران ، المملكة العربية السعودية

يناير ١٩٩٦م

Nomenclature

X	Value of the attribute of the product after being processed by machine. X is a random variable.
$\phi(x)$	Standard normal probability density function (pdf)
$\Phi(x)$	Standard normal cumulative probability density function (cdf)
L	Lower specification limit
A	Selling price of a good item
μ	Mean of the process setting
σ	Standard deviation
$t = \frac{\mu - L}{\sigma}$	standardized excess amount
N	Lot size
n	Sample size
D	Number of nonconforming items found in a sample of size n
d_0	Allowable number of nonconforming items found in a sample of size n

Chapter 1

Introduction

1.1 Quality: Statistical Quality Control

Quality is an important term nowadays. It is essential that products meet the requirements of those who use them. Therefore, quality is defined as *fitness for use* [23]. Every product possesses a number of elements that jointly describe its fitness for use. These parameters are often called *quality characteristics* . Quality characteristics may be of several types:

- **physical:** Length, weight, voltage, viscosity
- **Sensory:** Test, appearance, color
- **Time orientation:** Reliability, maintainability, serviceability

Most organizations find it difficult (and expensive) to provide the customer with products that have flawless quality characteristics. A major reason for this difficulty is *variability* [23]. There is a certain amount of variability in every product due to *randomness* of a process. Consequently no two products are ever identical. If variation is small, then it may have no impact on the customer. But if the variation is large, then the customer may perceive the unit to be undesirable and unacceptable. This is why reducing variability is an important consideration in quality control . *Statistical Quality Control* is a subject of study that use statistical technique to control variation of processes.

In reality, Statistical quality control technique in manufacturing and quality assurance have had a long history. In 1924 *Walter A. Shewhart* of the Bell Telephone Laboratories developed the statistical control chart concept. This is generally considered as the beginning of statistical quality control. World war II saw the widespread use and acceptance of statistical quality control concepts in manufacturing industries. Since then , almost all of the industries throughout the world have been using statistical process control techniques.

1.2 Control charts:

Control charts are widely used to maintain statistical control of a process. They are also effective devices for estimating process parameters and analyzing process capability. To use a control chart, the engineer must specify a sample size, a sampling frequency or interval between samples, and the control limits or critical region for the chart. Selection of these parameters is called the design of the control chart. There are several types of control charts described in literature, among those are control charts for attributes, control charts for variables, cumulative-sum and exponentially weighted moving-average control charts. A nice treatment of those charts is available in Montgomery [23].

1.3 Economics in statistical quality control

The design of a control chart has economic consequences in that the costs of sampling and testing, the costs associated with investigating out-of-control signals and possible correcting assignable causes, and costs of allowing defective products to reach the consumer are all affected by the selection of the control chart parameters. Therefore, it is logical to consider the design of a control chart from an economic viewpoint. In recent years, considerable attention has been devoted to this problem [23].

The economic design of an $\bar{X}$ chart began with the classic work of Duncan [11] who proposed a continuous expected cost model which determined sample size, sampling interval, and control limit by minimizing the expected cost per unit time of process operation. Due to the computational complexity of such models, they have been historically difficult to implement in practice. However, recent advances in computer processing technology, especially the availability of high speed processing technology on personal computers, has changed this according to Lorenzen and Vance [21], and such software has been developed and succesfully implemented. For example, Lorenzen and Vance describe the development of an economic chart for monitoring the carbon-silicate content of molten iron. They demonstrated that the use of such a chart vis-a-vis traditional non-economic charts may save an average of $\$600,000$ annually.

Another important economic aspect of quality control is finding the optimum target value (mean setting) of a processes (or a machine) . The general problem in this area is as follows:

Assume that we have a machine that processes a product. The product is assumed to have an attribute which is related to the processing of the product, and the material added to it by the machine (a product attribute could be weight, diameter, width, length, thickness, tensile strength, electrical resistance, etc.). The attribute has a lower specification limit (LSL) set for it. After being processed by the machine, we assume that the value of the attribute of the product is a random variable X. The product is accepted if $X \geq LSL$, and rejected otherwise. If the target is set at a high value then the material and processing costs are increased, but the rejection cost is less (due to the fact that the probability that a product is rejected becomes small). On the other hand, if the target value is set low then the material and processing costs are reduced, but the rejection cost becomes larger (due to the fact that the probability that a product is rejected becomes the more). Therefore, one has to set the target value at a point that strikes a compromise between the above conflicting objectives. Thus, the general problem is to determine the optimal value of μ that minimizes the costs of quality and production.

In this thesis, we will consider this kind of manufacturing processes and develop models to select optimum target values with several inspection criterias for processes with one and two machines.

1.4 Proposed work and objectives of the thesis

The objectives of the thesis are to study the optimal process settings for the case of a single machine and two machines with various inspection policies. The following are the goals of the thesis:

1. Determination of the optimum target value for a single filling process with rectifying inspection.

2. Determination of the optimum target values for the two machines in series with rectifying inspection.

3. Determination of the optimum target values for the two machines in series with 100% inspection.

4. Investigate cost savings due to reduction in machine variance on a single filling process for the case of rectifying inspection.

5. Developing an interactive Computer Package to solve various targeting models in the literature.

1.5 Organization

The notation used in common throughout the thesis has been described in nomenclature. The notation which is needed for specific chapters will be presented in the related chapters. A survey of literature in the field of application of the process targeting is given in chapter 2. Development of the single machine with rectifying inspection is given in chapter 3. Development of the two machines with rectifying inspection is presented in chapter 4. Two machines with 100% inspection is presented in chapter 5. Effect of the variance reduction on the process of a single machine with rectifying inspection is provided in chapter 6. An interactive computer package for process targeting is presented in chapter 7, followed by chapter 8 which includes contributions, conclusions and prospects for future research.

Chapter 2

Literature review

In this chapter we will give a brief overview of the literature review in the area of process targeting.

1. **Springer[32]** was the first to consider the problem of finding the optimal mean for a process with specified upper and lower limits, but he considered the cost of producing underfilled and overfilled cans as fixed.

2. **Nelson[26]** provide a nomograph for Springer's[32] model.

3. **Bettes[5]** solved the problem of choosing the optimal target value with upper and lower specification limits. However, his procedure is based on trial and error, is computationally tedious and does not give accurate values.

4. **Hunter and Kartha (HK) [19]** formulated a model where cans which are filled below a pre-specified limit L are sold in secondary market at a fixed price R. Cans field above L are sold in a regular market at a fixed price of A ($A > R$) per can. In one sense HK model is unrealistic because almost full cans are sold at same price as empty cans.

5. **Nelson[25] :** gave an approximating function of HK model having parameters not exceeding three significant figures.

6. **Bisgaard, Hunter, and Pallesen (BHP) [6]** extended HK model where cans filled below a prespecified limit L are sold in a secondary market at a price proportional to their fill at R per unit of weight. Cans field above L are sold in a regular market at a fixed price of A ($A > R$) per can.

7. **Vidal [35]** provided a simple graphical solution of BHP model.

8. **Carlsson (CL) [8]** extended the work of HK where the producer compensates the customer for underfilled cans and as well producer's benifit proportional to overfill . The amount of compensation is a fixed penalty plus an amount proportional to the difference between the lower specification limit and the actual fill .

9. **Golhar (GL) [13]** studied the targeting problem with the assumption that underfilled cans are to be emptied and refilled at the expense of the reprocessing cost.

10. **Golhar and Pollock (GP)[14]** GP extended the GL model for the case of finding an optimal upper limit and a mean process setting for uncapacited situation. As the upper limit approaches infinity, the GP model becomes GL. This model is same as Bettes [5] model but it is computationally easy and always give accurate solution.

11. **Golhar (GLv) [15]:** provided a computer program to solve GP model .

12. **Schmidt and Pfeifer (SPv) [29] :** have examined variance effects on Golhar's [13] models and provided an approximate close form solution of their developed model.

13. **Arcellus and Rahim (AR) [3] :** AR developed a model which incorporates the joint control of both variable and attribute quality characteristics of a product. Acceptability of an item hinges on succesful completion of specification tests associated with both quality characteristics. The tests are based on a lower specification limit for the varibales quality characteristics and an upper specification limit for the attribute quality characteristic.

14. **Schmidt and Pfeifer (SP) [30]:** SP extended GL work to include optimal upper limit and mean process setting for the case of the capacitated filling process.

15. **Boucher and Jafari (BJ) [7] :** BJ was the first who extended HK's work for the case of sampling inspection. A container is labeled as nonconforming if it is filled to less than the lower specification limit . If the number of nonconforming units found in the sample is greater than the allowable number of nonconforming units, the lot will be accepted otherwise rejected.

16. **Golhar and Pollock (GPv) [16]:** studied the cost savings due to variance reduction from GP model and provied approximate close form solutions.

17. **Al-Sultan and Al-Fawzan (AF) [2]:** AF examined variance effects on Rahim and Banerjee [28] model.

18. **Carlsson (CLv) [9] :** finds optimum process means considering two variable characteristics with acceptance sampling by variable.

19. **Al-Sultan (AS) [1] :** AS was the first to develop an algorithm to find optimum machine settings for the case of two machines in series with sampling inspection.

20. **Mihalko and Golhar (MG) [24] :** Usually process variance is an unknown parameter, but in most of the literature it is assumed to be given. Very recently, Mihalko and Golhar developed a scheme for the determination of the confidence interval for the optimal process setting for the case of an unknown process variance.

Chapter 3

Single machine with rectifying inspection

In this chapter we consider the Boucher and Jafari (BJ) model, and we extend their results for the case when a sampling plan is used with rectifying inspection.

3.1 Introduction

Rectifying inspection is commonly used in many industries ([11], [12], [20], [23]). We are motivated to consider rectifying inspection by the fact that rejected lots can have considerably different numbers of defective units, and it is unfair to sell them for the same reduced price as in the BJ model. Our proposed model may also be appropriate when there is no secondary market for rejected lots, in which case rejected lots have to be rectified by 100% inspection, and replacing (or reworking) defective items. Another way to look at the utility of this model is that it can be used to increase the value of the average outgoing quality of the produced lots. This is especially important in multistage manufacturing.

This chapter is organized as follows: in Section 3.2, we introduce some notation, Statement of the problem is presented in Section 3.3, and assumptions are stated in Section 3.4. Our model is presented in Section 3.5, followed by soution methodology in Section 3.6. Finally, an example is discussed in Section 3.7.

3.2 Notation

The following symbols are used in this chapter:

C=Cost of processing/unit of the product attribute.

A_1= Selling price/item for items in 100 percent inspected lots.

A_2= Selling price/item for items in lots accepted by acceptance sampling .

R_I= Reprocessing cost / item.

R_L= Reprocessing cost for all rejected items in a rejected lot.

I_C= Inspection cost/item

3.3 Statement of the problem

Consider a filling process characterized by a random variable X, which represents the quantity of material placed in an individual container. The lower specification limit for X is L. Containers from a production lot that is accepted under a certain acceptance sampling plan are sold at price A_2 per container. Rejected lots will undergo 100% inspection, and defective items will be reworked (or scrapped and replaced) thus making all the containers in the lot acceptable. These containers will be sold at a price A_1 [here $A_1 > A_2$] . This is realistic since those products which undergo 100% inspection should have a selling price higher than those products which are accepted under acceptance sampling. The sampling plan is performed as follows:

A container is labeled nonconforming if it is filled to less than the lower specification limit[$X < L$]. A sample of size n is drawn from every lot and evaluated. Let D be the number of nonconforming units found in the sample, and d_0 is the allowable number of nonconforming units. The lot will be accepted if $D \leq d_0$ and rejected if $D > d_0$. Figure 3.1 depicts the model for this problem.

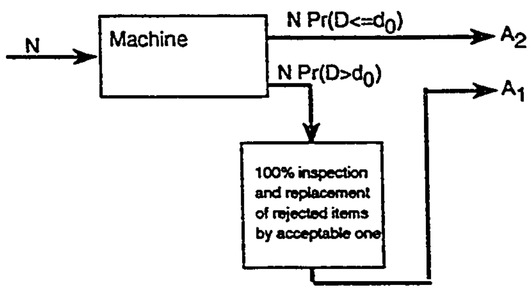

Figure 3.1: Quality targeting model for single machine with rectifying inspection

The problem is to find the optimal set point μ^* for the process such that total profit is maximized. The total profit is the sum of the revenues from selling containers minus the cost of processing and reprocessing and the cost of inspection for rejected lots.

3.4 Assumptions

1. X is normally distributed with mean μ and standard deviation σ.

2. $A_1 > A_2$

3. Let X_i represents the ith fill amount, then X_i is assumed to be independent of all X_j, j=1,2,......i-1.

4. A sampling plan is used where a sample of size n is taken and if the number of nonconforming units found in the sample is $> d_0$ the lot is rejected otherwise it is accepted (see [23] for a discussion on sampling plans).

5. Reworking is perfect.

6. The production of defective items are independent.

7. Processing cost is actually the cost of the ingredient.

8. No drift in the setting

3.5 The model

Given the above description of the problem, one could construct the following conditional expected profit functions:

$$\begin{aligned} E[R/D] &= A_2N - nI_C - NC\mu \qquad\qquad if D \leq d_0 \\ &= A_1N - R_L - NI_C - NC\mu \qquad\qquad if D > d_0 \end{aligned} \tag{3.1}$$

Where $E[R/D]$ is the expected profit for a lot of size N, given D.

As the production of defective items are independent, then the number of defective items in a lot of size N follows a binomial distribution. For a selected value of μ (the set point for the mean), let q be the probability of producing a defective item.

The total number of defective items in a random lot is equal to qN. However, the total number of defective items in a rejected lot is given by

$$d_{rl} = E(D|D > d_0) + q(N - n) \tag{3.2}$$

Where,

$q = Pr(X < L) = \Phi[-z]; z = \frac{\mu - L}{\sigma}$

$E(D|D > d_0)$=The expected number of defectives found in the sample, given that the lot was rejected

$q(N - n)$= The expected number of defectives in the non-sample portion of the lot.

From the theory of conditional probability and binomial distribution one can see that

$E(D|D > d_0)=\frac{\sum_{d=d_0+1}^{n} d\frac{(n)!}{d!(n-d)!}q^d(1-q)^{n-d}}{1-\sum_{d=0}^{d_0} \frac{(n)!}{d!(n-d)!}q^d(1-q)^{n-d}}$

Therefore, total repair cost of a rejected lot $=R_L=R_I[(d_{rl})]$

For the set point μ, the expected value of the processing cost per lot is $NC\mu$. Then the expected value of unconditional *marginal* profit for a lot of size N given that the set point is μ is given by

$$\begin{aligned} E[\mu] &= A_2NPr(D \leq d_0) - NC\mu \\ &\quad +[A_1N - R_L - (N - n)I_C]Pr(D > d_0) - nI_C \end{aligned} \tag{3.3}$$

Then the expected value of unconditional *marginal* profit per item given that the set point is μ is given by

$$
\begin{aligned}
E[\mu]/N &= A_2 Pr(D \leq d_0) - C\mu \\
&\quad + [A_1 - \frac{R_L}{N} - \frac{(N-n)}{N} I_C] Pr(D > d_0) - \frac{n}{N} I_C
\end{aligned}
\tag{3.4}
$$

where $Pr(D \leq d_0)$ is the probability that the number of nonconforming units (D) found in a sample of size n is less than or equal to the allowable number of defectives (d_0), and $Pr(D > d_0)$ is the probability that the number of nonconforming units(D) found in a sample of size n is greater than the allowable number of defectives (d_0).

3.6 Solution and analysis

To find the optimum process setting, the profit function above needs to be maximized. Then the best process setting is obtained from the relation $\mu^*=L + z^*\sigma$. Here, equation 3.4 depends on R_L which involves pdf and cdf of normal variates, so that an analytical solution is impossible. Therefore, a one dimensional search technique is needed for this problem. A review of these search techniques can be found in Bazaraa et al. [4] or any other classical book on nonlinear programming. One of these techniques is the Golden section method. In this technique, the initial search interval corresponds to the bounds on μ. Within this interval, two values are selected for evaluation. These values are chosen using the Golden section ratio. The profit is computed at each of these two values of μ and the interval is reduced according to the profit values at the given points. The process is repeated as many times as needed to get the required accuracy.

One needs to study the second derivative of the above function and ensures that it is negative everywhere to be sure that the function is concave (which ensures that the point obtained is actually a global maximum). However this is very difficult to do analytically. Therefore, we resorted to empirical investigation which does not show as a proof of concavity but at least gives some insights. An extensive numerical testing (we tested it for 50 variety of examples) has been carried out for the function 3.4, and indicated concavity of $E[\mu/N]$ over a wide range of its parameter values.

It is well known that the efficiency of direct search procedures is a function of the initial value selected to start the procedure. The developed model in this section can also be treated as an extension of HK model for the case of rectifying inspection. Nelson [25] developed an approximate close form solution for HK model. Due to the similarity with the HK model Nelson's [25] equation has used here in a slightly modified way as an initial solution of this model. We tested 50 variety of examples which shows convergence was more likely to occure when the suggsted starting solution is used.

Nelson gave an approximate close form solution of HK by following equation

$$t^* = 4.07 - 0.545Y - 0.054Y^2 - 10^{-2.73+0.712Y} \qquad (3.5)$$

where,

t^* is the optimum standardized excess amount

$$Y = 4 + log_{10}[z] \qquad (3.6)$$

$$z = \frac{C\sigma}{A - R} \qquad (3.7)$$

Actually Nelson's original equation has an extra term ϵ [$|\ \epsilon\ | < 0.0041$] whose value is determined by a graph provided by Nelson.

Here for equation (3.5) we will assume z (equation 3.7) values as follows:

$z = \frac{nC\sigma(A_1-A_2)}{R}$ if $d_0 = 0$

$z = \frac{17nC\sigma(A_1-A_2)}{R}$ if $d_0 = 1$

$z = \frac{17nC\sigma(A_1-A_2)(1-d_0/n)^2}{R}$ if $d_0 > 1$

3.7 Computational Experience

We illustrate the above model by solving a numerical example. For this we use data from the BJ model with additional values for A_1, I_C, N.

$A_1 = 80$

$A_2 = 67.5$

$C = 55/lb$

$R_I = 30.5$

$I_C = 1.0$

$L = 1.00lb$

$\sigma = 0.00563$

$n = 1$

$d_0 = 0$

$N = 100$

Using equation 3.5 we get suggested starting solution =1.0085. Golden section method has been used on equation 3.4 using suggested starting solution which gives optimum μ^* =1.0046 and corresponding profit per item 13.2630. Tables 3.1 and 3.2 are generated using Golden section search methods on equations 3.4 with the same data unless otherwise stated.

Explanation of notations used in table 3.1 and 3.2 are given below:

z=Standardized excess level=$\frac{\mu-L}{\sigma}$

q=The probability of an item being defective

$U = A_1 - R_I q - \frac{N-n}{N} I_C$

V =% increase of profit relative to basic model

For table 3.2:

DV=Parameter whose value deviates from that in case 1

Y=change in the value of DV from that in case 1

b=Basic model's parameter value

The example given at the begining of this Section will be treated as the 'Basic model'.

Table 3.1: Comparison of sampling plan effect on the single machine with rectifying inspection

	$n = 1$				$n = 10$				$n = 20$			
d_0	z	q	U	V	z	q	U	V	z	q	U	V
0	0.828	0.203	72.55	0	1.118	0.131	75.94	33.39	1.288	0.098	76.09	46.48
1					0.856	0.195	72.88	16.31	1.044	0.148	74.53	33.72
2					0.703	0.241	71.37	4.90	0.884	0.188	73.24	23.37
3					0.606	0.272	70.25	-2.15	0.766	0.221	72.13	14.66
4					0.540	0.294	69.37	-5.77	0.675	0.249	71.18	7.398
5					0.473	0.317	68.45	-7.16	0.606	0.272	70.08	1.59
6					0.381	0.351	67.25	-7.44	o.556	0.289	69.71	-2.71
7					0.242	0.404	65.51	-7.27	0.521	0.301	69.16	-5.58
8					0.050	0.479	63.15	-6.87	0.494	0.310	68.67	-7.22
9					-0.227	0.590	59.85	-6.27	0.469	0.319	68.18	-7.98
10									0.435	0.331	67.60	-8.24
11									0.378	0.352	66.81	-8.25
12									0.306	0.378	65.89	-8.13
13									0.223	0.411	64.78	-7.96
14									0.128	0.449	63.58	-7.74
15									-0.018	0.492	62.22	-7.49
16									-0.105	0.541	60.73	-7.22
17									-0.245	0.596	59.10	-6.89
18									-0.416	0.661	57.23	-6.51
19									-0.655	0.743	54.94	-5.98

Table 3.2: Sensitivity analysis for single machine with rectifying inspection

Case	DV	Y	z	q	V
1 [b]	Basic model		0.8282	0.2087	0
2	A_2	3	1.0048	0.1574	18.54
3		7	1.287	0.0989	44.85
4	A_1	-3	1.0048	0.1574	-4.07
5		-7	1.287	0.0989	-7.92
6	I_C	1	0.8848	0.1881	-1.53
7		2	0.9424	0.173	-2.96
8	R_I	5	0.930	0.1761	-1.40
9		9	1	0.158	-2.28
10	L	0.1	0.8282	0.2037	-41.46
11		0.2	0.8277	0.204	–82.93
12	σ	-.00463	0.8818	0.1889	1.64
13		0.00447	0.781	0.217	-1.49
14	N	100	0.823	0.205	0.217
15		-80	0.867	0.192	-1.703
16		-90	0.923	0.177	-3.74

From table 3.1, the entry ($n = 1$; $d_0 = 0$) is the solution for lot sampling when the sample size is 1, the lot size is large in relation to the sample size, and the number of nonconformities permitted is 0. As the sample size increases, the set point also increased. This result is to be expected because, for any value of q, the OC curve of a sampling plan with a higher n will show a smaller probability of acceptance. Put simply, sampling more units of a lot containing nonconforming units increases the likelihood of finding a nonconformity and rejecting the lot. This derives the producer to increase set point.

As the allowable number of nonconformities d_0 is increased, the optimum set point falls. This result is to be expected since raising the value of d_0 has the effect of enlarging the area of acceptance under the OC curve corresponding to the sampling plan. Simply stated, allowing more nonconformities to occure gives the producer more latitude in producing nonconformities.

One extra thing which can be seen from table 3.1 is that the increase or decrease in profit depends on the value of A_2N compared to $A_1N - R_IqN - (N-n)C_I$. If a lot is accepted, then the selling price for the lot is A_2N , while rejected lots will be sold for NA_1 but costs R_IqN for reworking defective items (since the lot will have qN defective items on the average) and cost $(N-n)C_I$ for inspection. Hence if $A_2 < A_1 - R_Iq - \frac{N-n}{N}C_I$ it is better that the lot be subjected to 100% screening because the cost of reworking defective items and inspection will be more than offset by the higher price for the 100% screened lot. In table 3.1, clearly $A_2 < A_1 - R_Iq - \frac{N-n}{N}C_I$ for all values of $d_0 = 0, 1, 2, ...11$ for sample size 20, and in these cases it is better to reject the lot as explained above. However, as d_0 increases, the probability of a lot being rejected gets smaller and that is why V gets smaller.

For $d_0 \geq 12$, $A_2 > A_1 - R_I q - \frac{N-n}{N} C_I$, and therefor it is better (more profitable) to accept the lot as explained above. It is also to be noticed that as d_0 increases, the probability of a lot being accepted gets higher, and that is why V gets larger with increasing values of d_0 after 12.

In addition, an extensive parametric analysis of the model was performed to investigate the effect of the various parameters $C, \sigma, I_C, L, R_I, A_1, A_2, L, N$ on the optimal target mean μ^*, as well as the percentage increase on the total expected profit per lot relative to the original example (here we call it basic model). Table 3.3 gives a sample of these results. The first case covers the example given above in section 3.7. The remaining cases also use the same set of values as case 1 does, except for the value of the parameter referred to in the DV, Y column.

The interpretation of the results is straightforward. When A_2 increases for the cases of 2 and 3 we get more profit. Here it has been observed that increasing A_2 will increase the mean process setting which ultimately lowers the probability of rejected lots, so that most of the lots should be accepted by the acceptance sampling plan to take advantage of the higher selling price which ultimately increases profit. If we reduce the price for 100% inspected items for the cases of 4 and 5, we will see that it decreases the profit. This will cause more lots to pass the acceptance sampling inspection by increasing the set point so that the probability of defectives is reduced. From the above explanation it is clear that if we increase the inspection cost for cases 6 and 7 the model will automatically drag the set point higher enabling more lots to pass the acceptance sampling inspection. However, this action will reduce the profit a little because of the increased processing cost.The same explanation is valid for cases 8 and 9 where the increasing repair cost will push the system away from more repair work by increasing the set point and thus lowering the probability of defectives. Increasing the lower specification limit will obviously cause a considerable loss of profit , because using more filling material without increasing the selling price will cause a drastic profit decrement. For this model, reducing variance for the case of 12 results in increased profit. Similarly, increasing the variance for the case of 13 obviously decreases profit. It happened here because we did not consider the cost for reducing machine variability. In practice, reducing machine variability costs a considerable amount of money which might offset the profit made by the reduction in variance . Finally, the smaller the lot size , the higher is the profit. This is because of inspection cost involved per item inspection. This is clear from the cases of 14-16.

From the above sensitivity analysis it is clear that for the cited example problem this model is sensitive to the parameter costs of processing material(C), lower specification limit(L) and the selling price of the acceptance sampling product(A_2). For other parameters, this model is moderately insensitive.

3.8 Summary

In this chapter, the problem of finding the optimal target mean value for a process with rectifying inspection has been considered. A mathematical model has been developed for this problem, and a solution procedure has been proposed to obtain the optimal set value. A numerical example has also been provided.

Chapter 4

Two machines in series with rectifying inspection

In this chapter, we extended the model developed in chapter 3 for the case of two stage manufacturing process . The model could also be considered as an extension of Al-Sultan's model [1] where rectifying inspection is used.

4.1 Introduction

Here, we assume that we have two machines in series that process a product. The product is assumed to have two attributes which are related to the processing of the product, and material added to it by machine 1 and machine 2 respectively (a product attribute could be weight, diameter, width, length, thickness, tensile strength, electrical resistance, etc.). Each attribute has a lower specification limit set for it, and if the measured value for that attribute is below its specification limit, the product is rejected. Primarily sampling inspection is done on lots produced from each machine. For the case of $1st$ machine, rejected lots have to be rectified by repairing or replacing whatever it requires. After being rectified, all the items from $1st$ machine are qualified for processing by the $2nd$ machine. Rejected lots from $2nd$ machine are also rectified in the same manner as machine 1. It is conceivable that those items which experienced 100% inspection will have higher price than those items which are accepted by sampling inspection.

Rectifying inspection is commonly used in many industries ([11], [12], [23]). The above model has application in multistage manufacturing industries. In many industries, in-process inspection is to be done in several stages of manufacturing and in many industrial situations, rejected lots (for in-process or finished goods) have no secondary market. Therefore, rejected lots have to be rectified by 100% inspection, and replacing (or reworking) defective items. We are also motivated to consider rectifying inspection by the fact that rejected lots could have considerably different number of defective units, and it is unfair to sell them for the same reduced price . Another way to look at the utility of this model is that it can be used to increase the value of the average outgoing quality of the produced lots. Gupta and Sagar[18], and Juran [19] have cited this kind of manufacturing processes where two stage rectifying inspection is applied. For example, in some rolling operations industries where preliminary preperation is accomplished on slitter which slot the blank to meet certain length (Juran [19]). After sampling inspection, those slotted lots fail to meet inspection criteria are sent for 100% inspection where defective items are replaced with acceptable one. Acceptable lots then proceed for further processing in the second stage in which then eloctroplating is done with certain thickness. Those electroplated roll's lot that fail to meet thickness specification limit from the sampling inspection will be rejected, and sent for 100% inspection where defective ones are replaced by acceptable ones (Fig 6.1).

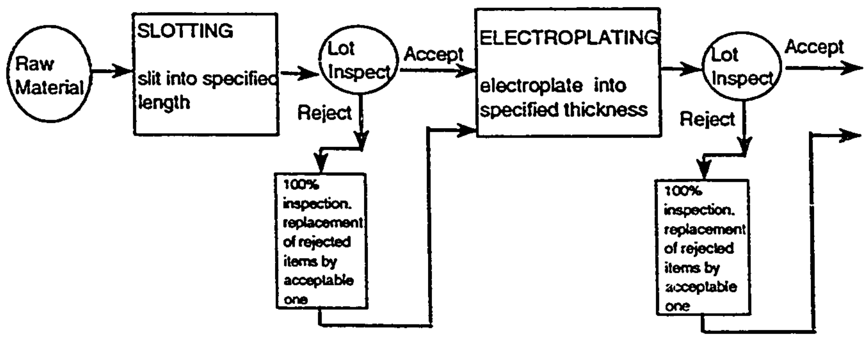

Figure 4.1: Sequence of processes in Sheet rolling industries

Often customer set a specified lower limit for both the roll length and the electroplating thickness . However any excess length and thickness are acceptable. From the manufacturers standpoint, excess roll length and eloctroplating thickness which is a result from higher process settings result in a proportionately larger give away cost. On the other hand, very tight process settings will have less processing cost, but at the expense of an increased rejection cost and eventually the cost of replacing defective ones. Therefore, it is desirable to set the process means for both machines optimally to minimize the total system cost. In this chapter, we address this problem and develop a model for finding the optimal process settings for the two machines with rectifying inspection.

This chapter is organized as follows: in Section 4.2, we introduce some notation, Statement of the problem is presented in Section 4.3, and assumptions are stated in Section 4.4. Our model is presented in Section 4.5, followed by soution methodology in Section 4.6. Finally, an example is given in Section 4.7.

4.2 Notation

The following symbols are used in this chapter:

$i = 1, 2$

X_i=Value of the ith attribute of the product after being processed by machine i (X_i is a random variable), $i = 1, 2$

L_i=Given lower specification limit for the ith product attribute , $i = 1, 2$

C_i=Cost of processing /unit of the ith product attribute , $i = 1, 2$

μ_i=Process mean setting for machine i , $i = 1, 2$

σ_i=Process variance setting for machine i , $i = 1, 2$

A_1= Selling price/item for items in lots by acceptance sampling

from both machine

A_2= Selling price/item for items in lots accepted by 100% inspection for $1st$ machine and acceptance sampling for $2nd$ machine

A_3= Selling price/item for items in lots accepted by acceptance sampling for $1st$ machine and 100% inspection for $2nd$ machine

A_4= Selling price/item for items in lots by 100% inspection for both machines.

R_I^i= Cost of replacing a defective item by an acceptable one after machine i, $i = 1, 2$

I_C^i= Inspection cost/item after machine i, $i = 1, 2$

R_L^i= Cost of replacing all rejected items found in a rejected lot after machine i, $i = 1, 2$

N= Lot size

n_i= Sample size for sampling after machine i, $i = 1, 2$

d_i= Allowable number of nonconforming items found in a sample of size n_i for machine i, $i = 1, 2$

D_i= Number of nonconforming items found in a sample of size n_i, $i = 1, 2$

4.3 Statement of the problem

Consider two machines in series. A lot of N items is sequentially processed by the two machines. After being processed by the first machine, we assume that the value of the first attribute of the product is a random variable X_1. After processing by the first machine, the lot is either rejected or accepted by sampling inspection. Rejected lots will undergo 100% inspection, where defective items will be reworked (or scrapped and replaced) which makes all the items acceptable to pass over to the second machine. After being processed by the second machine, the product's second attribute is assumed to be random variable X_2. Final products lot is either rejected or accepted by sampling inspection. Rejected lots will undergo 100% inspection again, and defective items will be reworked (or scrapped and replaced) which makes all the product acceptable. Given the above , four combination of lots is possible from two machines, obviously different combinations have different selling prices, these are:

1. A_1 : Selling price/item of the Acceptance sampling items for both machines
2. A_2 : Selling price/item of the 100% inspected items for the $1st$ machine and Acceptance sampling items for the $2nd$ machine
3. A_3 : Selling price/item of the Acceptance sampling items for the $1st$ machine and 100% inspected items for the $2nd$ machine

4. A_4 : Selling price/item of the 100% inspected items for both machines

Now products that have passed through 100% screening will have selling prices higher than those accepted by acceptance sampling , that means for the above case $A_4 > \{A_2, A_3\} > A_1$.

We assume that a sampling plan is implemented as follows: a lot of N products get processed by the first machine. Then a sample of size n_1 is taken from the lot, tested and if more than d_1 products have their first attribute $< L_1(D_1 > d_1)$, then the lot is rejected and will undergo 100% screening . After being processed by second machine, a sample of size n_2 is taken from the lot, tested and if more than d_2 products have their second attribute $< L_2(D_2 > d_2)$, then the lot is rejected and will undergo 100% screening ; otherwise the lot will be accepted. Figure 1 depicts the model for this problem.

The problem is to find optimal set point μ_1^* ,μ_2^* for the two machines such that the total profit is maximized. The total profit is the sum of the revenues from selling final products from machine 2 minus the cost of processing and reprocessing and cost of inspection for rejected lots from machine 1and 2.

4.4 Assumptions

1. X_1 and X_2 are normally distributed with mean μ_1 and μ_2 respectively and standard deviation σ_1 and σ_2 respectively.

2. Let X_K^i represents the ith fill amount on the Kth machine, then X_K^i is assumed to be independent from all X_K^j, j=1,2,......i-1 K=1,2. Moreover, X_1^i and X_2^i are also assumed to be independent for all i.

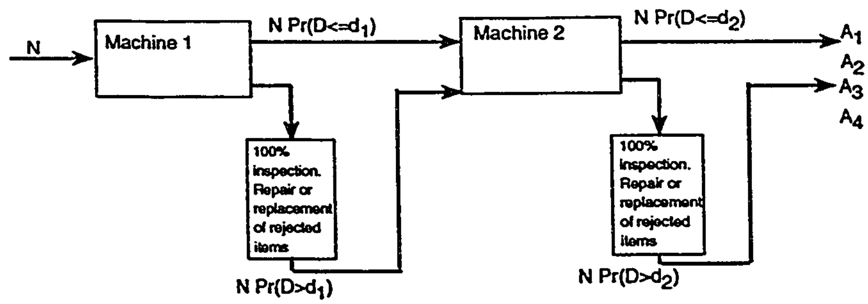

Figure 4.2: Quality targeting model for two machines in series with rectifying inspection.

3. $A_4 > \{A_2, A_3\} > A_1$

4. A sampling plan is used where a sample of size n_i , $i = 1, 2$ is taken and if the number of nonconforming units found in the sample is $> d_i$ for machine i, $i = 1, 2$ the lot is rejected otherwise it is accepted (see Montgomery [1991] for a discussion on sampling plans)

5. The machine sequence is fixed (i.e. products have to be processed by machine 1 first and then by machine 2 next).

6. The production of defective items are independent.

7. Reworking is perfect

8. Costs of processing is assumed to be directly proportional to the values of the product attributes.

9. No drift in the setting

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4.5 The model

As the production of defective items are independent, then the number of defective items in a lot of size N follows a binomial distribution. For a selected value of μ (the set point for the mean), let q be the probability of producing a defective item.

The total number of defective items in a random lot is equal to qN. However, the total number of defective items in a rejected lot is given by

$$d_{rl} = E(D|D > d_0) + q(N - n) \tag{4.1}$$

Where,

$q = Pr(X < L) = \Phi[-z]; z = \frac{\mu - L}{\sigma}$

$E(D|D > d_0)$=The expected number of defectives found in the sample, given that the lot was rejected

$q(N - n)$= The expected number of defectives in the non-sample portion of the lot.

From the theory of conditional probability and binomial distribution one can see that

$E(D|D > d_0) = \frac{\sum_{d=d_0+1}^{n} d\frac{(n)!}{d!(n-d)!} q^d (1-q)^{n-d}}{1 - \sum_{d=0}^{d_0} \frac{(n)!}{d!(n-d)!} q^d (1-q)^{n-d}}$

Therefore, total repair cost of a rejected lot $= R_L = R_I[(d_{rl})]$

For the set point μ_1, μ_2 the expected value of the processing cost per lot is $NC_1\mu_1$ and $NC_2\mu_2$ respectively. Then the expected value of unconditional *marginal* profit for a lot of size N given that the set point is μ_1, μ_2 is given by

$$\begin{aligned} E[\mu_1, \mu_2] &= -n_1 I_C^1 Pr(D_1 \leq d_1) \\ &\quad -(R_L^1 + NI_C^1) Pr(D_1 > d_1) \end{aligned}$$

$$
\begin{aligned}
&-n_2 I_C^2 Pr(D_2 \le d_2)\\
&-(R_L^2 + N I_C^2) Pr(D_2 > d_2)\\
&+A_1 N Pr(D_1 \le d_1) Pr(D_2 \le d_2)\\
&+A_2 N Pr(D_1 > d_1) Pr(D_2 \le d_2)\\
&+A_3 N Pr(D_1 \le d_1) Pr(D_2 > d_2)\\
&+A_4 N Pr(D_1 > d_1) Pr(D_2 > d_2)\\
&-N C_1 \mu_1 - N C_2 \mu_2 \qquad (4.2)
\end{aligned}
$$

One can rearrange $E[\mu_1, \mu_2]$ defined in equation 4.2 as follows.

$$
\begin{aligned}
E[\mu_1,\mu_2]/N \;=\; & -\frac{n_1}{N} I_C^1 - (R_L^1/N + \frac{(N-n_1)I_C^1}{N}) Pr(D_1 > d_1)\\
& -\frac{n_2}{N} I_C^2 - (R_L^2/N + \frac{(N-n_2)I_C^2}{N}) Pr(D_2 > d_2)\\
& +A_1 Pr(D_1 \le d_1) Pr(D_2 \le d_2)\\
& +A_2 Pr(D_1 > d_1) Pr(D_2 \le d_2)\\
& +A_3 Pr(D_1 \le d_1) Pr(D_2 > d_2)\\
& +A_4 Pr(D_1 > d_1) Pr(D_2 > d_2)\\
& -C_1 \mu_1 - C_2 \mu_2 \qquad (4.3)
\end{aligned}
$$

Where $Pr(D_i \le d_i)$, $i = 1, 2$, is the probability that the number of nonconforming unit (D_i) found in a sample of size n_i is less then or equal to allowable number of defective (d_i). And $Pr(D_i > d_i)$ is the probability that number of nonconforming units(D_i) found in a sample of size n_i is grater then allowable number of defectives (d_i).

4.6 Solution and analysis

To find the optimum process setting, the profit function above needs to be maximized. Then the best process setting is obtained from the relation $\mu^*=L + z^*\sigma$. Here, equation 4.3 depends on R_L which involves pdf and cdf of normal variates, so that an analytical solution is impossible. Therefore, a two-dimensional search technique is needed for this problem. A review of these search techniques can be found in Bazaraa et al. [4] or any classical reference on nonlinear programming. An efficient and widely used technique for a problem of this nature is the search technique of Hooke and Jeeves [see Bazaraa et al. [4]). Here the method of Hooke and Jeeves with discrete steps has been used . However, Hooke and Jeeves techniques using line search can also be used for this purpose. Hooke and Jeeves search using discrete steps starts with a local exploration in small steps around the starting values of μ_1 and μ_2. If the exploration is a success, i.e., the profit increases during local exploration, the step size is enlarged;. If the exploration is a failure, the step size is reduced. If a change of direction is necessary, the method starts all over again with a new pattern. The search is terminated when a stationary point is attained. The optimum process settings μ_1^* and μ_2^* are found from corresponding maximum profit.

One needs to study the Hessian of the function 4.3 and ensure that it is negative definite to be sure that the function is concave (which ensures that the point obtained is actually a global maximum). However this is very difficult to do analytically. Therefore, we resorted to empirical investigation which does not serve as a proof of concavity but at least gives some insights. An extensive numerical testing (we tested it for 50 different examples) has been carried out for the function 4.3, and indicated concavity of $E[\mu_1, \mu_2]/N$ over a wide range of its parameter values.

It is well known that the efficiency of direct surch procedures is a function of the initial value selected to start the procedure. We will use the following equations as a starting solution of this model:

$$\mu_i = L_i + 0.5\sigma_i \tag{4.4}$$

Our extensive computational experience (we tested it for 50 variety of examples) shows convergence was more likely to occure when the suggsted starting solution is used.

4.7 Computational Experience

We illustrate the above model by solving a numerical example with two machines and the following parameters.

$A_1 = 32$

$A_2 = 34$

$A_3 = 35$

$A_4 = 40$

$C_1 = 1/lb$

$C_2 = 2/lb$

$I_C^1 = 2$

$I_C^2 = 3$

$L_1 = L_2 = 8lb$

$R_I^1 = 6$

$R_I^2 = 17$

$\sigma_1 = \sigma_2 = 1$

$n_1 = 10$

$n_2 = 15$

$d_1 = 1$

$d_2 = 0$

$N = 100$

Using 4.4 we get suggested starting solutions $\mu_1 = 8.5$ and $\mu_2 = 8.5$. Hook and Jeeves method has used on equation 4.3 using suggested starting solution which gives optimum $\mu_1^* = 8.7142$,$\mu_2^* = 9.226$ and corresponding profit per item 3.8855. Tables 4.1, 4.2 and 4.3 are generated using Hook and Jeeves methods on equations 4.3 with the same data unless otherwise stated. Explanation of notations used in table 4.1, 4.2 and 4.3 are given bellow:

$i = 1, 2$

z_i=Standardized excess level=$\frac{\mu_i - L_i}{\sigma_i}$

q_i=The probability of an item being defective

V=%increase of profit relative to basic model

for table 4.3:

DV=Parameter whose value deviates from that in case 1

Y=change in the value of DV from that in case 1

b=Basic model's parameter value

Example given at the beginning of Section (4.7) will be treated as 'Basic model'

Study of sampling plan effect on the proposed model

Table 4.1: Sampling plan effect for the $1st$ machine

	$n_1 = 10$		$n_1 = 15$	
d_I	z_1	V	z_1	V
0	0.816	6.60	0.926	9.79
1	0.582	0	0.713	4.68
2	0.421	-5.004	0.556	0.023
3	0.304	-8.10	0.431	-4.04
4	0.210	-9.00	0.332	-7.32
5	0.109	-7.78	0.256	-9.60

Table 4.2: Sampling plan effect for the $2nd$ machine

	$n_2 = 10$		$n_2 = 15$	
d_2	z_2	V	z_2	V
0	1.177	-2.29	1.226	0
1	1.039	-5.02	1.081	-6.33
2	0.929	-0.249	1.013	-7.30
3	0.786	7.81	0.943	-3.82
4	0.621	16.88	0.846	1.85
5	0.443	26.40	0.732	8.29

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Table 4.3: Sensitivity analysis for two machine with rectifying inspection

Case	DV	Y	z_1	z_2	q_1	q_2	V
1^b	Basic model	——	0.595	1.239	0.275	0.107	0
1	A_1	.5	0.596	1.235	0.275	0.108	0.425
2		1	0.613	1.246	0.269	0.106	0.886
3	A_2	.5	0.547	1.257	0.281	0.104	1.923
4		1	0.569	1.291	0.284	0.098	4.055
5	A_3	.5	0.643	1.227	0.260	0.109	2.14
6		1	0.714	1.226	0.237	0.110	4.75
7	A_4	.5	0.524	1.29	0.300	0.117	8.97
8		1	0.473	1.16	0.317	0.123	18.46
9	I_C^1	0.5	0.651	1.236	.257	0.108	-10.50
10		1	0.734	1.251	0.231	0.105	-20.39
11	I_C^2	.5	0.589	1.26	0.277	0.103	-10.86
12		1	0.600	1.300	0.274	0.096	-21.49
13	R_I^1	2	0.752	1.253	0.226	0.104	-10.02
14		4	0.880	1.280	0.189	0.100	-17.13
15	R_I^2	2	0.594	1.279	0.276	0.100	-4.31
16		4	0.608	1.320	0.271	0.092	-8.55
17	σ_1	0.2	0.536	1.22	0.295	0.111	-2.88
18		-0.5	0.692	1.242	0.244	0.107	8.20
19	σ_2	0.2	0.571	1.17	0.283	0.120	-12.34
20		-0.5	0.619	1.36	0.267	0.085	33.35
21	N	-80	0.434	1.073	0.331	0.141	-15.30
22		300	0.618	1.264	0.268	0.103	3.75

From table 4.1 and 4.2, it is clear that as the sample size increases, the set point for the corresponding machine also increases and when allowable number of non-conformities is increased, the optimum set point falls which is exactly like chapter 3's model single machine with rectifying inspection. The explanation for this phinomena is just like the explanation given for the case of single machine in chapter 3.

In addition, an extensive parametric analysis of the model was performed to investigate the effect of various parameters $\sigma_i, I_C^i, R_I^i, A_1, A_2, A_3, A_4, N$ on the optimal target means μ_1^*, μ_2^*, as well as the percentage increase in the total expected profit per lot relative to the original example (here we call it basic model). Table 4.3 gives a sample of these results.The first case covers the example given above in section 4.7. The remaining cases also use the same set of values as those of case 1, except for the value of the parameter referred to in the DV, Y column.

The interpretation of the results from table 4.3 is straight forward. It has been observed that by increasing the cost of processing, the process mean decreases a little bit . When the selling price of products accepted by acceptance sampling (A_1) increases for the cases of 2 and 3, we get more profit. Here it has been observed that increasing A_1 will increase mean process setting of both machine which ultimately lowers the probability of rejected final lots, so that most of the lots will be accepted by acceptance sampling plan to take advantage of higher selling price which ultimately increase profit. If we increase price for 100% inspected items (A_4) for the cases of 8 and 9, we will see that it also increases the profit. Here it has been observed that increasing A_4 will decrease mean process setting for both machines which ultimately increases the probability of the rejection of final lots, so that most of the lot should go for 100% inspection and take the advantage of higher price (A_4) which ultimately increase profit. Cases (4-7) are easily explainable using the arguments sated for the cases of (2,3 and 8,9). From the above explanation it is clear that if we increase inspection cost for the cases 10-13 the model will automatically drag the set point higher for the corresponding machine to make more lots pass acceptance sampling inspection. However this action will reduce the profit a little because of the increase in processing cost. The same explanation is valid for cases 14 - 17, where increasing repair cost will push the system not to go for more repair work which is possible by increasing the set point for the corresponding machine and thus lowering the probability of defectives. For this model, increasing variance for the cases of 17 and 19 causes decreased profit similarly decreasing variance for the cases of 19 and 21 increases profit which is intuitive. It happened here, because we did not consider the cost for reducing machine variability. However in real life , reducing machine variability costs some money which might offset the profit made by the reduction in variance . Here , the smaller the lot size , the lower is the profit. It is because of the bigger sample size to lot size ratio. Higher the sample size to lot size ratio lower will be the profit. This is clear from the cases of 21 and 22.

From the above sensitivity analysis it is clear that for the cited example problem this model is sensitive to the parameters of cost of processing material(C_i), Standard deviation(σ_i) the selling price of products (A_4) . For other parameters, this model is moderately insensitive.

4.8 Summary

In this chapter, the problem of finding the optimal target mean value for two machines in series with rectifying inspection has been considered. A mathematical model has been developed for this problem, and a solution procedure has been proposed to get the optimal set value. A numerical example has also been provided.

Chapter 5

Two machines in series with 100% inspection

In this chapter, we extended Golhar's model [13] for the case of two stage manufacturing process. This work can also be thought of as a modified version of Al-Sultan's[1] model when 100% inspection is used.

5.1 Introduction

Researchers have undertaken several studies on the optimum targeting for a process considering recycling of rejected products and other factors. Over this area Golhar[13] studied the case in which a rejected product would be recycled so that it would be sold in the primary market. Golhar and Pollock[14] determined optimal process mean and optimal upperlimit for recycling products. Recently Gupta and Golhar [17] has been determined both the optimal process mean and optimal lot size for the rejected recycled products.

For all the above cases researchers have cited examples from pharmaceuticals industries [13] and glass industries [17] showing only a single stage operation. However, there are some industrial situations with two stages. For example, in some glass tube industries raw material is melted and rolled into sheets with certain thickness [31]. After inspection, those sheets which fail to meet thickness specification limit are crashed and recycled from the beginning. Those that pass inspection, proceed for further processing in the second stage in which tubes with certain length are made out of these sheets. Those tubes that fail to meet length specification limit will be rejected, and melted for reprocessing right from the beginning . Due to technological constraints rejected final products cannot be reprocessed from intermediate stage (Fig. 5.1). Often customer set a specified lower limit for both the sheet thickness and the tube length . However any excess thickness and length are acceptable. From the manufacturers standpoint, excess sheet length and thickness which is a result from higher process settings result in a proportionately larger give away cost. On the other hand, very tight process settings will have less processing cost, but at the expanse of an increased rejection cost and eventually the cost of recycling and reprocessing. Therefore, it is desirable to set the process means for both machines optimally to minimize the total system cost. In this paper, we address this problem and develop a model for finding the optimal process settings for the two machines. The model developed here will also have application in paper and plastic industries where multistage recycling is performed.

This chapter is organized as follows: in Section 5.2, we introduce some notation, Statement of the problem is presented in Section 5.3, and assumptions are stated in Section 5.4. Our model is presented in Section 5.5, followed by soution methodology in Section 5.6. Finally, an example is given in Section 5.7.

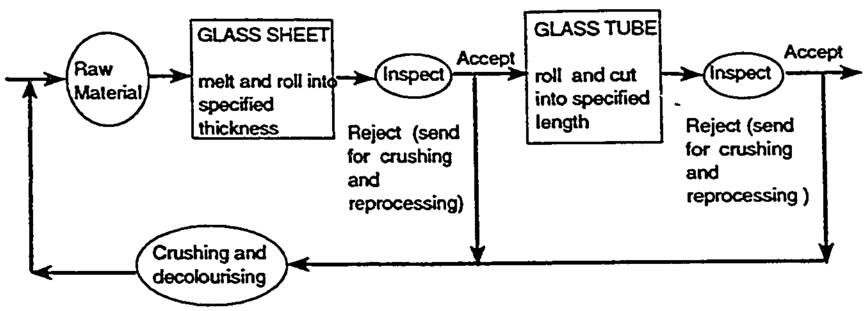

Figure 5.1: Sequence of processes in manufacturing Glass Tubes

5.2 Notation

The following symbols are used in this paper:

$i = 1, 2$

X_i=Value of the attribute of the product after being processed by machine i (X_i

is

a random variable), $i = 1, 2$

L_i=Given lower specification limit for the ith product attribute , $i = 1, 2$

C_i=Cost of processing /unit of the ith product attribute , $i = 1, 2$

μ_i=Process mean setting for machine i , $i = 1, 2$

σ_i=Process variance setting for machine i , $i = 1, 2$

A= Selling price/item for accepted item

R_i= Reprocessing cost / item after machine i , $i = 1, 2$

5.3 Statement of the problem

Assume that we have two machines in series that process a product. The product is assumed to have two attributes which are related to the processing of the product, and material added to it by machine 1 and machine 2 respectively (a product attribute could be weight, diameter, width, length, thickness, tensile strength, electrical resistance, etc.). Each attribute has a lower specification limit set for it. After being processed by the first machine, we assume that the value of the first attribute of the product is a random variable X_1. After processing by the first machine, the item is either rejected or accepted by inspection. Rejected items will be recycled from the beginning at a fixed reprocessing cost R_1 (this might include the cost of production time lost, among others). After being processed by the second machine, the product's second attribute is assumed to be random variable X_2. Final items are either rejected or accepted. Rejected items will be recycled from the beginning at a fixed reprocessing cost R_2 . Figure 5.2 depicts the model for this problem.

If the process mean μ_1 and μ_2 for machine 1 and machine 2 respectively are set too high (well above the minimum weight specification) then the probability that the process will turn out a product with one of its attribute less than the LSL set for it becomes very small, thus avoiding reprocessing cost. This saving will be at the expense of selling overdesigned products at a fixed regular price, in which case the company bears the cost of excess quality. On the other hand, if the process mean μ_1 and μ_2 are set too low, then the company will save on the give-away cost, but with the expense of an increase in reprocessing cost (see Nelson[25]). The problem is to find the optimal process mean setting μ_1^* (for machine 1) and μ_2^* (for machine 2) that will maximize an appropriate measure of the expected profit.

The total profit is the sum of the revenues from selling final products minus the cost of processing and reprocessing by the two machines.

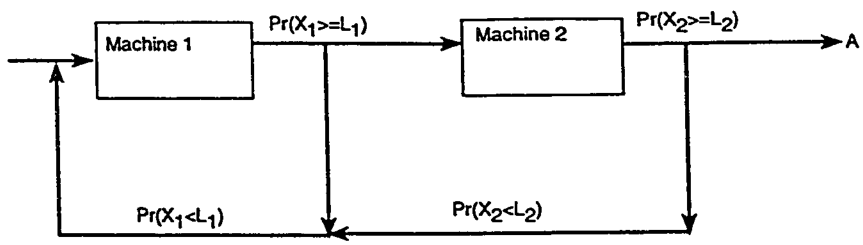

Figure 5.2: Quality targeting model for two machines in series with 100% inspection

5.4 Assumptions

1. X_1 and X_2 are normally distributed with mean μ_1 and μ_2 respectively and standard deviation σ_1 and σ_2 respectively.

2. Let X_K^i represents the *ith* fill amount on the Kth machine, then X_K^i is assumed to be independent from all X_K^j, j=1,2,......i-1 K=1,2. Moreover, X_1^i and X_2^i are also assumed to be independent for all i.

3. The production of rejected items are independent.

4. The machine sequence is fixed (i.e. products have to be processed by machine 1 first and then by machine 2 next).

5. Costs of processing is assumed to be directly proportional to the values of the product attributes.

6. No drift in the setting

5.5 The model

For the set point μ_1 and μ_2 the expected value of the processing cost per item is $C_1\mu_1$ and $C_2\mu_2$ respectively. Let $P(X_1, X_2, \mu_1, \mu_2)]$ be the profit for a can fielled with contents X_1 and X_2, and $E[P(X_1, X_2, \mu_1, \mu_2)]$ be the expected profit. If an item has one of its two attributes less than lower specification limit set for it , it is reprocessed at the cost of R_1 (if it is rejected from machine 1) and R_2 (if it is rejected from machine 2) . This reprocessed item will then realize the expected profit $E[P(X_1, X_2, \mu_1, \mu_2)]$. Given the above description of the problem, one could construct the following profit function per can :

$$
\begin{aligned}
P(x_1, x_2, \mu_1, \mu_2) &= A - C_1x_1 - C_2x_2 && If \;\; x_1 \geq L_1 \;\; and \;\; x_2 \geq L_2 \\
&= E[P(X_1, X_2, \mu_1, \mu_2)] - R_1 && If \;\; x_1 < L_1 \;\; and \;\; x_2 \geq L_2 \\
&= E[P(X_1, X_2, \mu_1, \mu_2)] - R_2 && If \;\; x_1 \geq L_1 \;\; and \;\; x_2 < L_2 \\
&= E[P(X_1, X_2, \mu_1, \mu_2)] - R_1 - R_2 && If \;\; x_1 < L_1 \;\; and \;\; x_2 < L_2
\end{aligned} \tag{5.1}
$$

Hence the expected profit is given by

$$
\begin{aligned}
&E[P(X_1, X_2, \mu_1, \mu_2)] = \\
&\quad \int_{L_1}^{\infty}\int_{L_2}^{\infty}(A - C_1X_1 - C_2X_2)g(X_1; \mu_1, \sigma_1^2)g(X_2; \mu_2, \sigma_2^2)dx_1dx_2 \\
&\quad + \int_{0}^{L_1}\int_{L_2}^{\infty}(E[P(X_1, X_2, \mu_1, \mu_2)] - R_1)g(X_1; \mu_1, \sigma_1^2)g(X_2; \mu_2, \sigma_2^2)dx_1dx_2 \\
&\quad + \int_{0}^{L_1}\int_{L_2}^{\infty}(E[P(X_1, X_2, \mu_1, \mu_2)]] - R_2)g(X_1; \mu_1, \sigma_1^2)g(X_2; \mu_2, \sigma_2^2)dx_1dx_2 \\
&\quad + \int_{0}^{L_1}\int_{0}^{L_2}(E[P(X_1, X_2, \mu_1, \mu_2)] - R_1 - R_2)g(X_1; \mu_1, \sigma_1^2)g(X_2; \mu_2, \sigma_2^2)dx_1dx_2
\end{aligned} \tag{5.2}
$$

5.6 Solution and analysis

In this section, we propose a procedure for solving the model developed in the last section.One can rearrange $E[\mu_1, \mu_2]$ defined in (5.1) as follows.

$$
\begin{aligned}
E[P(X_1, X_2, \mu_1, \mu_2)] &= A(1 - \Phi[-z_1])(1 - \Phi[-z_2]) \\
&- \int_{L_1}^{\infty} \int_{L_2}^{\infty} (C_1 X_1 + C_2 X_2) g(X_1; \mu_1, \sigma_1^2) g(X_2; \mu_2, \sigma_2^2) dx_1 dx_2 \\
&+ (E[P(X_1, X_2, \mu_1, \mu_2)] - R_1)\Phi[-z_1](1 - \Phi[-z_2]) \\
&+ (E[P(X_1, X_2, \mu_1, \mu_2)] - R_2)(1 - \Phi[-z_1])\Phi[-z_2] \\
&+ (E[P(X_1, X_2, \mu_1, \mu_2)] - R_1 - R_2)\Phi[-z_1])\Phi[-z_2]
\end{aligned} \tag{5.3}
$$

where,

$$z_i = \frac{\mu_i - L_i}{\sigma_1} \qquad i = 1, 2 \tag{5.4}$$

$$q_i = Pr(X_i < L_i) = \Phi[-z_i] \qquad i = 1, 2 \tag{5.5}$$

rearranging equation 5.3 we get

$$
\begin{aligned}
E[P(X_1, X_2, \mu_1, \mu_2)] &= A\Phi[z_1]\Phi[z_2] \\
&- \int_{L_1}^{\infty} \int_{L_2}^{\infty} (C_1 X_1 + C_2 X_2) g(X_1; \mu_1, \sigma_1^2) g(X_2; \mu_2, \sigma_2^2) dx_1 dx_2 \\
&+ (E[P(X_1, X_2, \mu_1, \mu_2)] - R_1)\Phi[-z_1]\Phi[z_2] \\
&+ (E[P(X_1, X_2, \mu_1, \mu_2)] - R_2)\Phi[z_1]\Phi[-z_2] \\
&+ (E[P(X_1, X_2, \mu_1, \mu_2)] - R_1 - R_2)\Phi[-z_1])\Phi[-z_2]
\end{aligned} \tag{5.6}
$$

where,

$$1 - \Phi[-z_i] = \Phi[z_i] \qquad i = 1, 2 \tag{5.7}$$

using the following result from Golhar [1987]

$$\int_L^\infty X g(X; \mu, \sigma^2) dx = (\mu\Phi[z] + \sigma\phi[-z]) \tag{5.8}$$

we get the following equation from 5.6

$$\begin{aligned} E[P(X_1, X_2, \mu_1, \mu_2)] &= A - C_1\mu_1 - C_2\mu_2 - \frac{1}{\Phi[z_1]\Phi[z_2]}(C_1\sigma_1\Phi[z_1]\phi[z_1] \\ &\quad + C_2\sigma_2\Phi[z_2]\phi[z_2] + R_1\Phi[-z_1] + R_2\Phi[-z_2]) \end{aligned} \tag{5.9}$$

Clearly one would like to find μ_1^*, μ_2^* that maximizes $E[P(X_1, X_2, \mu_1, \mu_2)]$ defined in 5.9 above. To maximize the function 5.9, we will use its derivative information .

A necessary condition for optimality is that the partial derivative with respect to μ_1 and μ_2 vanishes at the target value μ_1^* and μ_2^*.

Taking pertial derivative of 5.9 with respect to μ_1 and equating to zero we get following equations:

$$\frac{\partial E[P(X_1, X_2, \mu_1^*, \mu_2^*)]}{\partial \mu_1} = 0 \tag{5.10}$$

$$\begin{aligned} \frac{\partial E[P(X_1, X_2, \mu_1^*, \mu_2^*)]}{\partial \mu_1} &= -C_1 - \frac{1}{\Phi[z_1]\Phi[z_2]}(C_1\sigma_1\Phi[z_1]\partial_1 - C_1\sigma_1\delta_1 + R_1\delta_1) \\ &\quad -(C_1\sigma_1\Phi[z_1]\phi[z_1] + C_2\sigma_2\Phi[z_2]\phi[z_2] \\ &\quad + R_1\Phi[-z_1] + R_2\Phi[-z_2])\delta_{11}^1 \end{aligned} \tag{5.11}$$

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Where,

$$\partial_1 = \frac{\partial \phi[-z_1]}{\partial \mu_1} = -\frac{z_1}{\sigma_1}\phi[z_1] \tag{5.12}$$

$$\delta_1 = \frac{\partial \Phi[-z_1]}{\partial \mu_1} = -\frac{1}{\sigma_1}\phi(-z_1) \tag{5.13}$$

$$\delta_{11} = \frac{\partial}{\partial \mu_1}\{\frac{1}{\Phi(z_1)}\} = -\frac{\phi[z_1]}{\sigma_1 \Phi[z_1]^2} \tag{5.14}$$

$$\delta_{11}^1 = \frac{\partial}{\partial \mu_1}\{\frac{1}{\Phi(z_1)\Phi(z_2)}\} = -\frac{\phi[z_1]}{\sigma_1 \Phi[z_1]^2 \Phi[z_2]} \tag{5.15}$$

Taking pertial derivative of 5.9 with respect to μ_2 and equating to zero we get following equations:

$$\frac{\partial E[P(X_1, X_2, \mu_1^*, \mu_2^*)]}{\partial \mu_2} = 0 \tag{5.16}$$

$$\begin{aligned}\frac{\partial E[P(X_1, X_2, \mu_1^*, \mu_2^*)]}{\partial \mu_2} &= -C_2 - \frac{1}{\Phi[z_1]\Phi[z_2]}(C_2\sigma_2\Phi[z_2]\partial_2 - C_2\sigma_2\phi[z_2]\delta_2 + R_2\delta_2) \\ &\quad -(C_1\sigma_1\phi[z_1]\Phi[z_1] + C_2\sigma_2\phi[z_2]\Phi[z_2] \\ &\quad +R_1\Phi[-z_1] + R_2\Phi[-z_2])\delta_{12}^2\end{aligned} \tag{5.17}$$

Where,

$$\partial_2 = \frac{\partial \phi[-z_2]}{\partial \mu_2} = -\frac{z_2}{\sigma_2}\phi[z_2] \tag{5.18}$$

$$\delta_2 = \frac{\partial \Phi[-z_2]}{\partial \mu_2} = -\frac{1}{\sigma_2}\phi(-z_2) \tag{5.19}$$

$$\delta_{12} = \frac{\partial}{\partial \mu_2}\{\frac{1}{\Phi(z_2)}\} = -\frac{\phi[z_2]}{\sigma_2 \Phi[z_2]^2} \tag{5.20}$$

$$\delta_{12}^2 = \frac{\partial}{\partial \mu_2}\{\frac{1}{\Phi(z_1)\Phi(z_2)}\} = -\frac{\phi[z_2]}{\sigma_2 \Phi[z_2]^2 \Phi[z_1]} \tag{5.21}$$

We need to solve equation 5.11 and 5.17 in the two unknowns μ_1^*, μ_2^*. However, these two equations are not explicit in the unknowns, which make the task of solving them a little harder. There are several well-known numerical methods that can be used to solve the system of equations 5.11 and 5.17. A review of these numerical methods may be found in Bazaraa et al. [4] or other classical book on nonlinear programming. Here we have used RTBIS subroutine (based on the principle of bisection method) from Press et al. [27]. The idea for bisection method is simple. Over some interval the function is known to pass zero because its sign. Evaluate the derivative function at the intervals midpoint and examine each iteration the bounds containing the root decrease by a factor of two. For this problem, one starts with a given value for the target means, (say $\mu_1 = L_1 + \sigma_1$) or any other appropriate value and finds the root of equation 5.11 to yield a value for μ_2. Then one uses this value of μ_2 in equation 5.17 to find a new value for μ_1. One continues this process until the desired level of convergence for 5.9 has been reached. In addition, these results have been validated through the use of Hook and Jeeve's [see Bazaraa et al. [4]) method, a direct search technique on 5.9 that does not require the use of derivatives.

One needs to study the Hessian of the function 5.9 and ensure that it is negative definite to be sure that the function is concave (which ensures that the point obtained is actually a global maximum). However this is very difficult to do analytically. Therefore, we resorted to empirical investigation which does not serve as a proof of concavity but at least gives some insights. An extensive numerical testing (we tested it for 50 different examples) has been carried out for the function 5.9, and indicated concavity of $E[P(X_1, X_2, \mu_1^*, \mu_2^*)]$ (also see figure 5.3) over a wide range of its parameter values.

It is well known that the efficiency of direct surch procedures is a function of the initial value selected to start the procedure. We will use following equations as a starting solution of this model:

$$\mu_i = L_i + 0.5\sigma_i \tag{5.22}$$

Our extensive computational experience (we tested it for 50 variety of examples) shows convergence was more likely to occure when the suggsted starting solution is used.

5.7 Computational Experience

We illustrate the above model by solving a numerical example with two machines and the following parameters.

$A = 40$

$C_1 = 1/lb$

$C_2 = 2/lb$

$L_1 = L_2 = 8lb$

$R_1 = 6$

$R_2 = 17$

$\sigma_1 = \sigma_2 = 1$

Using 5.11, 5.17 and 5.9 we get $\mu_1^* = 9.63825$,$\mu_2^* = 9.759375$, profit per item $= 9.49052$.

Several observations with respect to our computational experience are worth mentioning. One of them relate to the relationship between the profit objective function and the target means and can be explained with the help of Figure 3 for the example problem. This figure shows that $E[P(X_1, X_2, \mu_1, \mu_2)]$ is concave function , which in turn, means that the local maximum obtained by setting the derivatives to zero is actually a global maximum. We conjecture that $E[\mu_1, \mu_2]$ is always concave although it is a bit involved to prove it analytically.

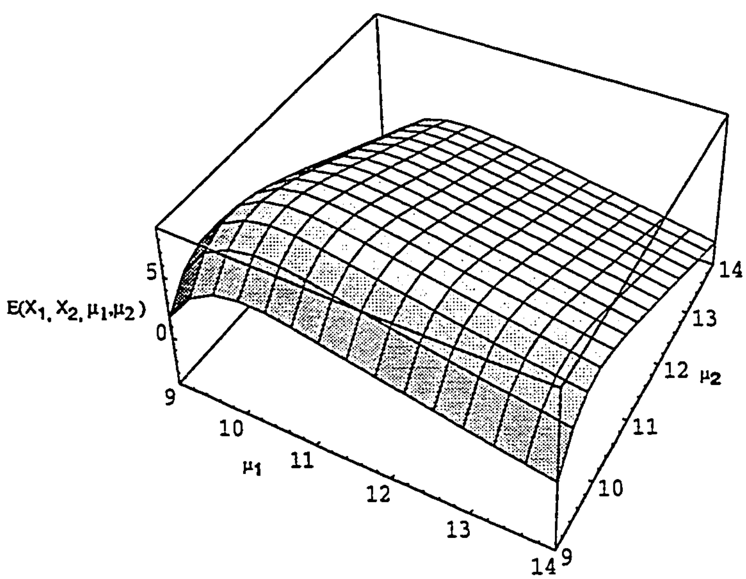

Figure 5.3: Expected value $E[P(X_1, X_2, \mu_1, \mu_2)]$ vs. target means (μ_1, μ_2).

Moreover some other extensive parametric analysis of the model was performed to investigate the effect of various parameters C_i, σ_i, R_i, on the optimal target means μ_1^*, μ_2^*, as well as the percentage increase in the total expected profit per lot relative to the original example (here we call it basic model). Table 5.1 gives a sample of these results. The first case covers the example given above in section 5.7. The remaining cases also use the same set of values as those of case 1, except for the value of the parameter referred to in the DV, Y column. Table 5.1 is generated using equations (5.8- 5.16) with the same data unless otherwise stated. Example given at the begining of Section (5.7) will be treated as 'Basic model' Explanation of notations used in table 5.1 is given bellow:

$i = 1, 2$

z_i=Standardized excess level=$\frac{\mu_i - L_i}{\sigma_i}$

q_i=The probability of an item being defective

V=%increase of profit relative to basic model

DV=Parameter whose value deviates from that in case 1

Y=change in the value of DV from that in case 1

b=Basic model's parameter value

Table 5.1: Sensitivity analysis for the two machines in series with 100% inspection

Case	DV	Y	z_1	z_2	q_1	q_2	V
1^b	Basic model	——	2.211	1.937	0.013	0.026	0
2	C_1	0.2	1.5476	1.7671	.0608	0.386	-20.46
3		0.3	1.506	1.772	0.066	0.038	-30.63
4	C_2	0.2	1.647	1.714	0.049	0.043	-20.71
5		0.3	1.651	1.693	0.049	0.045	-31.04
6	R_1	2	1.761	1.752	.039	0.039	-1.02
7		4	1.854	1.748	0.031	0.040	-1.822
8	R_2	2	1.636	1.809	0.051	0.035	-0.856
9		4	1.632	1.854	0.051	0.031	-1.625
10	σ_1	0.2	1.547	1.767	0.060	0.038	-3.60
11		-0.5	2.959	1.738	0.025	0.041	9.845
12	σ_2	0.2	1.656	1.672	0.048	0.047	-7.63
13		-0.5	1.592	2.070	0.055	0.019	20.75
14	$Ł_1$	2	1.638	1.759	.0506	0.039	-21.07
15		4	1.638	1.759	.0506	0.039	-42.14
16	$Ł_2$	2	1.638	1.759	.0506	0.039	-42.14
17		4	1.638	1.759	.0506	0.039	-84.29

The interpretation of the results from table 5.1 is straight forward. Cases 2 - 5 explore the effect of increasing the cost of processing/unit of the product attribute. One can clearly see that if the cost of processing/ unit increases then (without increasing the selling price/ unit) the profit is reduced. It has also been observed that by increasing the cost of processing, the process mean decreases a little bit . For cases 6 - 9, where increasing repair cost will push the system not to go for more repair work which is possible by increasing the set point for the corresponding machine and thus lowering the probability of defectives. For this model, increasing variance for the cases of 10 and 12 causes decreased profit similarly decreasing variance for the cases of 11 and 13 increases profit which is intuitive. It happened here, because we did not consider the cost for reducing machine variability. However; in real life , reducing machine variability costs some money which might offset the profit made by the reduction in variance . The Cases (14-17) consider the effect of changes in the specification limits on the optimal target means. Increasing the lower specification limits (L_1, L_2) causes to increase in set points with large variation of profit . However z_1^* appears to be independent of changes in L_1, and E_2^* appears to be independent of changes in L_2 due to the fact that z_1^* and z_2^* are functions of the standardized quantities $\frac{\mu_1^*-L_1}{\sigma_1}$ and $\frac{\mu_2^*-L_2}{\sigma_2}$ respectively, in which the incrase in L_1 is offset by the same improvement of increase in μ_1^*, with the same interpretation applies to μ_2^* and L_2.

From the above sensitivity analysis it is clear that for the cited example problem this model is sensitiveto parameters of cost of processing material(C_i), Standard deviation(σ_i) and Specification Limits (L_i) . For other parameters, this model is moderately insensitive.

5.8 Summary

In this chapter, the problem of finding the optimal target mean value for two machines in series with 100% inspection has been considered. A mathematical model has been developed for this problem, and a solution procedure has been proposed to get the optimal set value. A numerical example has also been provided.

Chapter 6

Improving process capability by variance reduction

In this chapter effect of variance on the performance of a single filling operation for the case of a rectifying process is investigated.

6.1 Introduction

In all of the models cited so far in the literature review (chapter 2), variance of the process is assumed to be constant and uncontrollable. In many industrial processes, controlling the mean process settings by itself (which is achieved by the above models) may not enough to reduce the expected cost per good item to an acceptable level. Because the new quality-management philosophies place particular emphasis on improving quality through improvements in process capabilities, it is equally important to examine the benefits associated with reductions in the process variance in addition to the benefits associated with setting the process mean. For example, one can see the cost effect of reducing variance from manual control to automatic control system in a plastic coating industry (Figure 6.1 , Twombly and Whiteman [33]) . Reduced variability of an automatic control system will save raw material . This saving will have to be compared with the cost of adopting the costly automatic control system (for other systems, variance reduction may be attained by training operators, quality of raw material etc.). Therefore, one has to do cost-benefit analysis to decide whether the automatic control system would be cost effective or not.

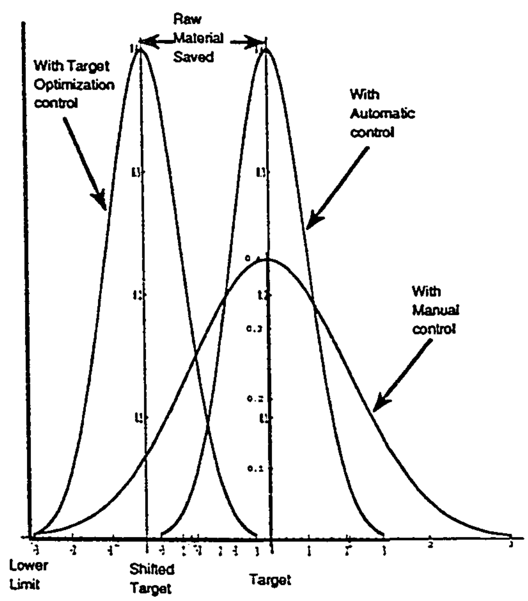

Figure 6.1: Material Saving by Variance Reduction

Three of the relevant works are Al-sultan and Al-fawzan [2], Golhar and Pollock[16], Schmidt and Pfeifer[29]. These researcher have examined variance effects on Rahim and Banerjee [28], Golhar and Pollock's [14] and Golhar's [13] models respectively.

In this chapter, we use the same approach of Golhar and Pollock[16] to study the effect of the variance on the expected total profit per good item for a single filling operation with rectifying inspection . A model for this process has been developed in chapter 3.

The chapter is organized as follows: in Section 6.1, we introduce a brief presentation of chapter 3 's developed model for the case of a single machine with rectifying inspection , and description of cost savings due to variance reduction is stated in Section 6.2. Finally, computational experience is discussed in Section 6.3.

6.2 Process targeting for a single machine with rectifying inspection

In the single filling operation problem described in chapter 3 , the process is characterized by a random variable X, which represents the quantity of material placed in an individual container. The lower specification limit for X is L. Containers from a production lot that is accepted under a certain acceptance sampling plan are sold at price A_2 per container. Rejected lots will undergo 100% inspection, and defective items will be scrapped and replaced thus making all the containers in the lot acceptable. These containers will be sold at a price A_1 [here $A_1 > A_2$] . A container is labeled nonconforming if it is filled to less than the lower specification limit[$X < L$]. A sample of size n is drawn from every lot (of size N) and evaluated. Let D be the number of nonconforming units found in the sample, and d_0 is the allowable number of nonconforming units per lot of size N. The lot will be accepted if $D \leq d_0$, and rejected if $D > d_0$.

Assume X is normally distributed with mean μ, and standard deviation σ and reworking is perfect then from the above description of the problem, one could construct the following conditional expected profit functions:

$$
\begin{aligned}
E[R/D] &= A_2N - nI_C - NC\mu \qquad if D \leq d_0 \\
&= A_1N - R_L - NI_C - NC\mu \qquad if D > d_0
\end{aligned} \tag{6.1}
$$

Where $E[R/D]$ is the expected profit for a lot of size N, given D.

For the set point μ, the expected value of the processing cost per lot is $NC\mu$. Then the expected value of unconditional *marginal* profit per item given a mean set point μ and a variance σ is given by

$$
\begin{aligned}
E[\mu]/N &= A_2Pr(D \leq d_0) - C\mu \\
&\quad +[A_1 - \frac{R_L}{N} - \frac{(N-n)}{N}I_C]Pr(D > d_0) - \frac{n}{N}I_C
\end{aligned} \tag{6.2}
$$

where

$$
Pr(D > d) = 1 - \sum_{d=0}^{d} \frac{(n)!}{d!(n-d)!} q^d (1-q)^{n-d} \tag{6.3}
$$

$$
\begin{aligned}
q &= Pr(X < L) \\
&= \Phi[-z] \\
z &= \frac{\mu - L}{\sigma}
\end{aligned} \tag{6.4}
$$

$$
R_L = R_l[(d_{rl})] \tag{6.5}
$$

$$d_{rl} = E(D|D > d_0) + q(N - n) \tag{6.6}$$

$$E(D|D > d_0) = \frac{\sum_{d=d_0+1}^{n} d\frac{(n)!}{d!(n-d)!}q^d(1-q)^{n-d}}{1-\sum_{d=0}^{d_0}\frac{(n)!}{d!(n-d)!}q^d(1-q)^{n-d}} \tag{6.7}$$

$E(D|D > d_0)$=The expected number of defectives found in the sample, given that the lot was rejected

This allows easy determination of the optimal process mean

$$\mu^* = L + z^*\sigma \tag{6.8}$$

Assuming that one can optimally set the process mean using (6.8), the mean becomes a function of the variance. It is then of interest to determine the cost savings as a function of the variance, as we do in the next section.

6.3 The value of improving Process Performance

Variance reduction could have effects on costs savings in two ways. First, a reduction in variance (without re-setting the mean) will directly reduce the amount of rejected items, and consequently the amount of rework needed. A second and less obvious cost saving is realized when the process mean is readjusted in light of the new lower variance. From equation (6.8), we can see that optimal mean decreases as σ is reduced. Thus, a small variance allows the process to be operated closer to the lower specification limit, with a subsequent reduction in the amount of material used.

We will use the approach by Golhar and Pollock[16] to study the effect of σ on the expected total cost per good item. Let us define the following function

$$Q(\sigma) \; = \; E[\mu|\sigma = 0] - E[\mu|\sigma > 0] \tag{6.9}$$

where

$Q(\sigma)$: is the expected excess minimum cost per good item due to the variance.

$E[\mu|\sigma > 0]$: is the expected profit per good item, for a certain value of $\sigma > 0$, when setting the mean at its corresponding optimal setting μ^*

$E[\mu|\sigma = 0]$: is the expected profit per good item when $\sigma = 0$

When $\sigma = 0$, then the problem is deterministic, leading to the trivial solution $\mu^* = L$. The resulting profit is $A_1 - CL$ per good item. Therefore $Q(\sigma)$ is the expected excess profit that one expects to pay per good item due to the increase in the value of σ from $\sigma = 0$ to its current value. $Q(\sigma)$ is given as follows

$$\begin{aligned} Q(\sigma) \; &= \; A_1 - CL - \{A_1 - C\mu^* + (A_2 - A_1)Pr(D \leq d_0) \\ & \quad -[R_I q + \frac{N-n}{N} I_C] Pr(D > d_0) - \frac{n}{N} I_C\} \\ &= \; C(\mu^* - L) + \frac{n}{N} I_C + (A_1 - A_2)Pr(D \leq d_0) \\ & \quad +[R_I q + \frac{N-n}{N} I_C] Pr(D > d_0) \end{aligned} \tag{6.10}$$

If the process standard deviation is reduced from σ to σ', the fractional saving (FS) can be defined as

$$FS = 1 - \frac{Q(\sigma')}{Q(\sigma)} \tag{6.11}$$

$Q(\sigma')$ and $Q(\sigma)$ can be easily obtained from equation (6.10), once the corresponding optimal mean settings are known. A flow chart for computing FS is provided in the Appendix.

6.4 Computational Experience

We consider the example given in chapter 3 with following datas

$A_1 = 80$

$A_2 = 67.5$

$C = 55/lb$

$R_I = 30.5$

$I_C = 1.0/item$

$L = 1.00lb$

$n = 5$

$d_0 = 1$

$N = 100$

The plot of the expected excess profit per unit good item $E[\mu, \sigma]$ as a function of σ and μ is shown in Figure 6.2. Clearly $E[\mu, \sigma]$ is concave and as σ increases, its maximum is attained at larger values of μ which is to be expected if one is to minimize rejects and consequently cost of rework.

The plot of the expected excess cost per unit good item $Q(\sigma)$ as a function of σ is shown in Figure 6.3. From Figure 6.3 It is clear that as σ increases from 0 , the expected excess minimum cost per unit good item increases sharply, and later it levels off in a *diminishing return fashion.* This is expected, since the effect of change in σ gets less important as σ increases in value.

Figure 6.4 shows $1 - \frac{Q(\sigma')}{Q(\sigma)}$, the fractional savings achieved by the fractional improvement of process variance (FV) ($FV = 1 - \frac{\sigma'}{\sigma}$) where $\sigma = 0.22$ for the example problem. From figure 6.4, it is clear that as fractional improvement of process variance FV increases, the fractional saving will increase very slowly, but soon it will increase sharply as FV reaches its maximum. This result agrees with the Figure 6.3.

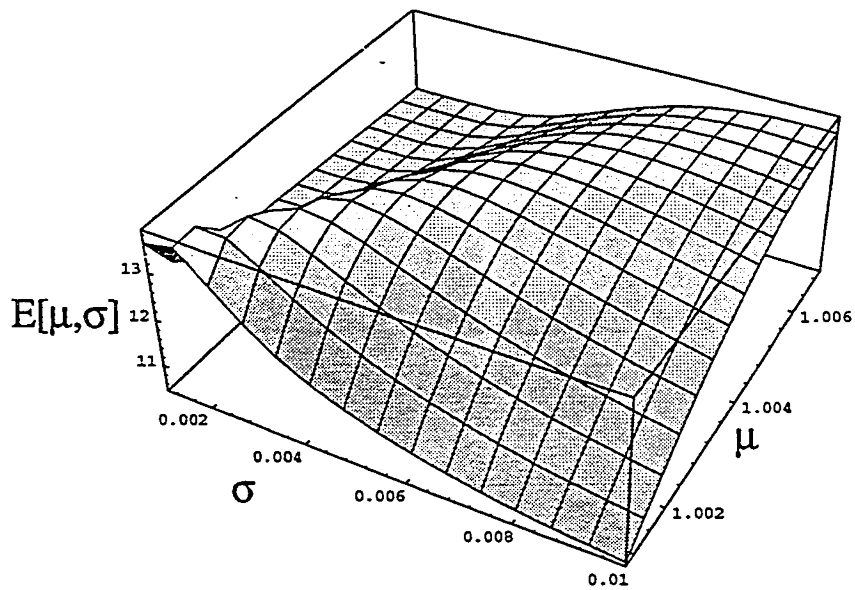

Figure 6.2: Plot of $E(\mu, \sigma)$ as a function of σ and μ

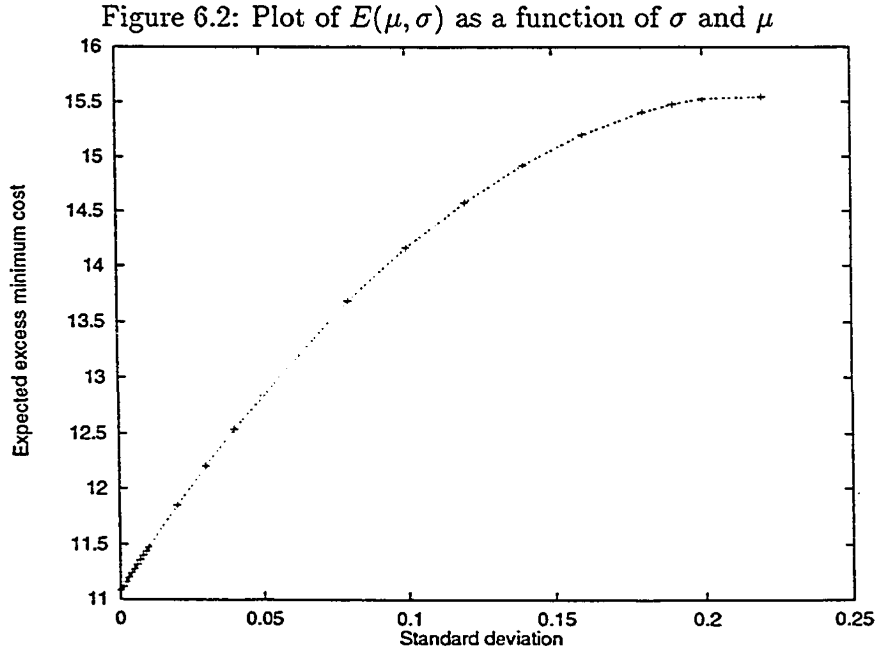

Figure 6.3: Plot of $Q(\sigma)$ as a function of σ

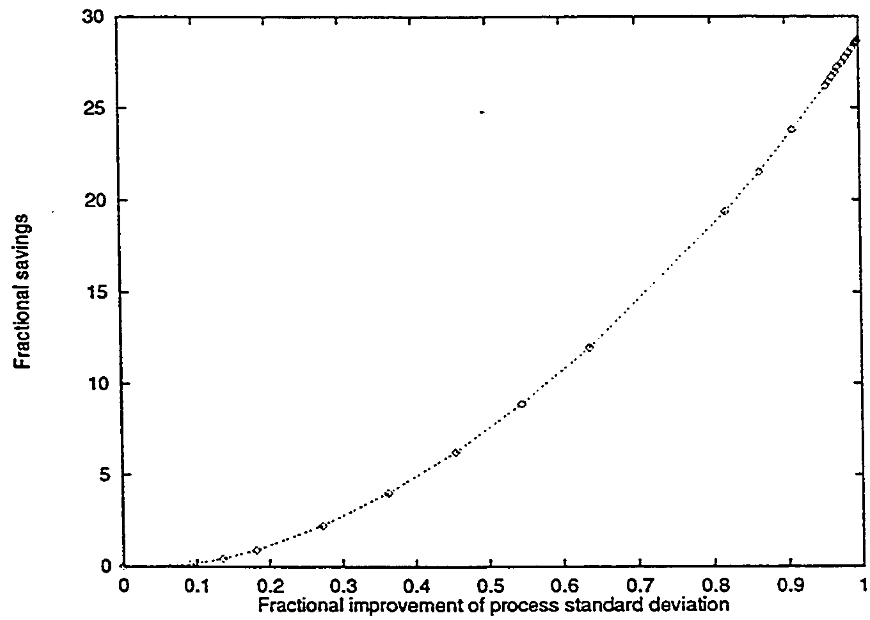

Figure 6.4: Fractional savings ($FS = [1 - \frac{Q(\sigma')}{Q(\sigma)}]100\%$) due to fractional improvement of process standard deviation ($FV = 1 - \frac{\sigma'}{\sigma}$) . Where σ is the present standard deviation and σ' is the improved process standard deviation

6.5 Summary

In this chapter, we have considered the effect of variance on the performance of a single filling operation. A function that measures the cost saving due to process variance reduction has been proposed. This cost saving should be compared with the cost of reducing the variance (which could be attained by training operators, regulating current for the machine, acquiring raw material with better quality, etc.), and a decision should be made to go for this reduction only if the saving outweighs the costs.

Chapter 7

A computer package for process mean targeting

In this chapter, we develop a computer package which includes 10 of the most important targeting models in the literature. The models considered for the computer package are following:

1. **Hunter and Kartha (HK) [19]**
2. **Bisgaard, Hunter, and Pallesen (BHP) [6]**
3. **Carlsson (CL) [8]**
4. **Golhar (GL) [13]**
5. **Schmidt and Pfeifer (SP) [30]**
6. **Boucher and Jafari (BJ) [7]**
7. **Pulak and Al-Sultan** [1]

[1]Pulak and Al-Sultan model is actually the model developed in chapter 3 of this thesis.

8. **Golhar and Pollock (GP) [14]**

9. **Arcelus and Rahim (AR) [3]**

10. **Al-Sultan (AS) [1]**

7.1 Introduction

Many researchers have recently considered the targeting of process mean under various conditions. They have developed several models for solving this problem. But none of the developed models provides a closed form solution, which means that iterative methods have to be used to solve these models. Most of the time the user is supposed to start with an initial solution and either a trial and error procedure or a numerically-based method is used to arrive at the optimal solution. This raises two difficult issues for general users, namely, to guess a good starting solution and to design the iterative procedure for obtaining the optimal solution. This suggests that there is a great need to develop a unified computer package that will solve these developed models. The package is written in FORTRAN, is interactive in nature, and requires minimal effort from the user. It will also provide a good initial solution for every model. However, there is no guarantee that this starting solution will always be the best. That is why, if the user wishes to begin with another guess the package will allow him to do so.

The chapter is organized as follows: in Section 7.2, we discuss obtaining a good starting solution. In section Section 7.3, we discuss program description, followed by program operation in Section 7.4. We present examples in Section 7.5. The code is provided in the appendix C.

7.2 Obtaining a starting solution

It has been observed that solution procedures are sensitive to initial solutions. This is why the user should be careful in picking an initial solution such that the procedure converge quickly. For a good starting solution following adaptation has done:

1. **Hunter and Kartha (HK) [19]** : For HK model we will use Nelson[25] 's approximate close form solution 3.5 as a starting solution.

2. **Bisgaard, Hunter, and Pallesen (BHP) [6]**: The initial solution for this model is the same as the HK model.

3. **Carlsson (CL) [8]** : The initial solution for this model is the same as the HK model.

4. **Golhar (GL) [13]** : For GL we will use authors approximate close form solution as starting solution which is as following:

$$t^* = -0.712 - 0.471 ln(\frac{R}{C\sigma}) \tag{7.1}$$

5. **Schmidt and Pfeifer (SP) [30]:** For SP we will use authors approximate close form solution as starting solution which is as following:

$$\begin{aligned} t_2^* = {} & Exp(0.000881Q^{-1} + 0.446 - 0.203Q - 0.0295Q^2 \\ & +0.00104Q^3) - 1.57 \end{aligned} \tag{7.2}$$

where, $t_2 = \frac{L-\mu}{\sigma}$

$$Q = \frac{A + R - CL}{C\sigma}$$

6. **Boucher and Jafari (BJ) [7] :** For BJ we will use Nelson's equation 3.5 in a slightly modified way as an initial solution of this model. Here for equation 3.7 we will assume z (equation 2.4) value as follows:

$z = \frac{nC\sigma}{R}$ if $d_0 = 0$

$z = \frac{17nC\sigma}{R}$ if $d_0 = 1$

$z = \frac{17nC\sigma(1-d_0/n)^2}{R}$ if $d_0 > 1$

7. **Pulak and Al-Sultan (PS) :** We will use Nelson's equation (3.5) in a slightly modified way as an initial solution of this model. Here for equation (3.7) we will assume z (equation 2.4) values as follows:

$z = \frac{nC\sigma(A_1 - A_2)}{R}$ if $d_0 = 0$

$z = \frac{17nC\sigma(A_1 - A_2)}{R}$ if $d_0 = 1$

$z = \frac{17nC\sigma(A_1 - A_2)(1-d_0/n)^2}{R}$ if $d_0 > 1$

8. **Golhar and Pollock (GP) [14] :** For GL model we will use authors developed initial solutions for this model which is as following:

$$t_2^* = -0.746\frac{R}{C\sigma} \tag{7.3}$$

$$t_1^* = -\ 2t_2^* \tag{7.4}$$

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where,

$$t_1 = \frac{U - \mu}{\sigma}$$

$$t_2 = \frac{L - \mu}{\sigma}$$

9. **Arcelus and Rahim (AR) [3] :** AR's suggestion for a good initial solution has used for this model.

   $\mu = (L + U)/2$ and $\lambda = l$

10. **Al-Sultan (AS) [1] :** For AS model at first the optimum process setting of machine 2 will be solved. We will use Nelson's equation (2.2) as an initial solution for this purpose. Here for equation (2.2) we will assume a z (here $z = z_2$ in equation 2.4) value as follows:

    $z_2 = \frac{nC_2\sigma_2}{A_3 - A_2}$ if $d_0 = 0$

    $z_2 = \frac{17nC_2\sigma_2}{A_3 - A_2}$ if $d_0 = 1$

    $z_2 = \frac{17nC_2\sigma_2(1-d_2/n_2)^2}{A_3 - A_2}$ if $d_0 > 1$

    After obtaining optimum process setting for machine 2, the following equation needs to be evaluated.

    $FX = C_2\mu_2^* + A_1 - A_2 + (A_2 - A_3)Pr(D_2 \leq d_2)$

    If $FX \geq 0$ stop, the problem is infeasible, i.e. use machine 1 only. If $FX < 0$ then we need to solve for the optimum process setting for machine 1. We will use again Nelson's equation (2.2) as an initial solution for this purpose. Here for equation (2.2) we will assume z(here $z = z_1$ in equation 2.4) value as follows:

$$z_1 = \frac{nC_1\sigma_1}{FX} \qquad \text{if } d_0 = 0$$

$$z_1 = \frac{17nC_1\sigma_1}{FX} \qquad \text{if } d_0 = 1$$

$$z_1 = \frac{17nC_1\sigma_1(1-d_1/n_1)^2}{FX} \qquad \text{if } d_0 > 1$$

7.3 Effectiveness of the starting solution

Nelson's [25] equation has been used to get a good starting solution for model 1 and for models 2, 3, 6, 7 and 10 with slight modification. From our extensive computational experience, we have observed that this suggested starting solution for model 1 works very well. For model 2 and 3 it also works reasonably well. For model 6 and 7, though it does not always work very well, it still gives user a feeling of how to arrive at a very good starting solution. For model 10, the user should carefully investigate the suggested starting solution and exploring with an alternative starting solution also, is always recommended.

For models 4, 5, 8 and 9 we have used the corresponding authors' approximate equations as suggested initial solution. In fact, for model 4, 5, and 8 they work very well. For model 9, they also work well but sophisticated users are recommended to start with some other smart solution if they can guess it in advance.

7.4 Program description

1. **Stage 1:** The package consists of a main calling program and several subroutines and function subprograms. The main program prompts the user to identify which model he wants to work with. After selecting the model, the user will be asked to write the values of the parameters of the chosen model, and the value of the initial starting solution.

2. **Stage 2:** In this package, we use derivative equations of profit function to get an optimum process setting for the case of model 10. For model 10, the root from the derivative equations are found by calling subprogram 2 and 3. The optimum process setting value is obtained by maximizing profit functions for the case of models 1 through 9. With the input of parameter values for the chosen model it will call either subprogram 6 or 7. In this program, models 1 through 7 call subprogram 6 and models 8 and 9 call subprogram 7.

   In subprogram 6, we have used a one dimensional search to maximize the profit function by the Golden section search method (see Bazaraa et al. [4]). In subprogram 7, the two dimensional direct search algorithm of Hook and Jeeves (see Bazaraa et al. [4]) is used. A search in both the subroutines will be terminated when the number of iterations equals a predetermined value or has achieved a prespecified accuracy, whichever occurs first. In the program listing we have given a brief description of each subprogram.

   A flowchart of this package is shown in Fig. 7.1 .

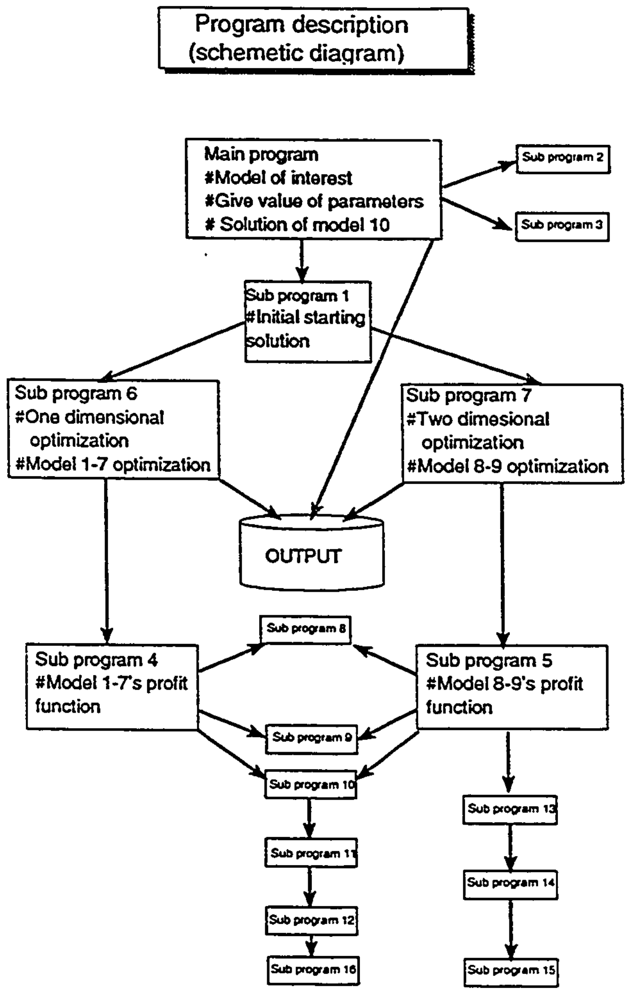

Figure 7.1: Program flow chart

7.5 Program operation

The program prompts the user for:

1. a selection of the model of interest;

2. the value of the selected model parameters; and

3. the initial value of the starting solutions.

The output may be sent to a screen or stored in a user given input file. The package is written in FORTRAN and the source code is provided in the program listing in Appendix C.

7.6 Examples

1. Example 1: for the purpose of illustration we are using data from the HK model, where R = \$37; C = \$55; A = \$67.5; $\sigma = 0.00563$ *lb* ;L=1 *lb*. The output of the program is, optimum standardized excess level = 2.710365, profit per item =\$ 66.557922, mean of the process setting=1.015259 *lb*.

2. Example 2: for the case of the model developed in chapter 3 (we named here as Pulak and Al-Sultan [1995] model) , we are using the same data from chapter 3, where $A_1 = 80\$$; $A_2 = 67.5\$$; $R = 30.5\$$; $C = 55\$/lb$; $\sigma = 0.00563$; $L = 1lb$; $N = 100$; $n = 10$; $d_0 = 1$; $C_I = 1\$$ per item. The output of the program is , optimum standardized excess level = 0.856106 ; profit per item =15.629\$; mean of the process setting= 1.00482*lb*. The output of examples 1 and 2 are given in output listing 1 and 2 respectively.

7.7 Conclusion

In this chapter, a FORTRAN based computer package is presented for solving various targeting models for process means which are available in the literature. Suggestions for good starting solutions have been presented and numerical examples have been provided. This program has been tested in using Microsoft compiler version 5.10 on IBM 433 DX personal computer and on Sun workstation. This program is sensitive to machine precision.

7.8 Output Listing 1:

```
PROCESS TARGETING PACKAGE


 WHICH MODEL DO YOU WANT?

 WRITE ====

        1 --- FOR HUNTER AND KARTHA

        2 --- FOR BISGAARD ET. AL

        3 --- FOR CARLSSON

        4 --- FOR GOLHAR

        5 --- FOR SCHMIDT & PFEIFER

        6 --- FOR BOUCHER & JAFARI

        7 --- FOR PULAK & AL-SULTAN

        8 --- FOR GOLHAR & POLLACK

        9 --- FOR ARCELUS AND RAHIM

        10--- FOR AL-SULTAN
1
 SELLING PRICE FOR REJECTED ITEMS
37
 GIVE AWAY COST PER UNIT WEIGHT
55
 SELLING PRICE OF AN ACCEPTED ITEM
67.5
 STANDARD DEVIATION OF THE PROCESS
0.00563
 LOWER SPECIFICATION LIMIT
1

 DO YOU WANT OUTPUT SENT TO THE  SCREEN OR TO A FILE?
```

```
        WRITE (S) FOR SCREEN,(F) FOR FILE
s

 SUGGESTED STARTING MEAN SETTING  IS:       1.015645623


 DO YOU WANT TO CONTINUE WITH SUGGESTED STARTING VALUE?
      WRITE (Y) FOR YES AND (N) FOR NO
y


 PROCESS TARGETING PACKAGE




 MODEL NO. =    1


 STANDARDIZED EXCESS LEVEL =  2.710365

 PROFIT PER ITEM  = 66.557922

 MEAN OF THE PROCESS SETTING=     1.015259
```

7.9 Output Listing 2:

```
 PROCESS TARGETING PACKAGE


 WHICH MODEL DO YOU WANT?

 WRITE ====

        1 --- FOR HUNTER AND KARTHA

        2 --- FOR BISGAARD ET. AL

        3 --- FOR CARLSSON

        4 --- FOR GOLHAR

        5 --- FOR SCHMIDT & PFEIFER

        6 --- FOR BOUCHER & JAFARI

        7 --- FOR PULAK & AL-SULTAN

        8 --- FOR GOLHAR & POLLACK

        9 --- FOR ARCELUS AND RAHIM

        10--- FOR AL-SULTAN
7
 SAMPLE SIZE
10
 ALLOABLE NO. OF DEFECTIVES
1
 SELLING PRICE OF 100% SCREENED  CAN
80
  SELLING PRICE OF ACCEPTENCE  SAMPLING PRODUCT
67.5
  COST OF INSPACTION PER ITEM FOR  REJECTED LOT
1
  LOT SIZE
```

```
100
 REPROCESSIG COST OF A REJECTED ITEM
30.5
 PER UNIT COST OF FILLING INGREDIENT
55
 STANDARD DEVIATION OF THE PROCESS
.00563
 LOWER SPECIFICATION LIMIT
1

 DO YOU WANT OUTPUT SENT TO THE  SCREEN OR TO A FILE?
        WRITE (S) FOR SCREEN,(F) FOR FILE
s

 SUGGESTED STARTING MEAN SETTING  IS:       1.006515980


 DO YOU WANT TO CONTINUE WITH SUGGESTED STARTING VALUE?
      WRITE (Y) FOR YES AND (N) FOR NO
y


 PROCESS TARGETING PACKAGE




 MODEL NO. =    7


 STANDARDIZED EXCESS LEVEL =  0.856106

 PROFIT PER ITEM  = 15.629005

 MEAN OF THE PROCESS SETTING=     1.004820
```

Chapter 8

Conclusions and recommendation for future research

In this thesis, the following problems have been investigated :

1. In chapter 3, the optimum target value for a single filling process with rectifying inspection has determined. A mathematical model has been provided for this problem , and a solution procedure has been proposed to obtain the optimal set value. A numerical example has also been provided.

2. In chapter 4, the optimum target values for the two machines in series with rectifying inspection has determined , and a solution procedure has been proposed to obtain the optimal set value. A numerical example has also been provided.

3. In chapter 5, the optimum target values for the two machines in series for 100% inspection has determined , and a solution procedure has been proposed to obtain the optimal set value. A numerical example has also been provided.

4. In chapter 6, cost savings due to reduction in machine variance on the model developed in chapter 3 has investigated. A function that measures the cost saving due to process variance reduction has been proposed.

5. In chapter 7, a FORTRAN based computer package is presented for solving various targeting models for process means which are available in the literature. Suggestions for good starting solutions have been presented and numerical examples have been provided. Complete code is provided in appendix C.

Upon concluding, the following are some areas for future research:

1. Determination of optimal lot size parameters and a process mean for the case of a rectifying inspection. It will be an extension of the model developed in chapter 3. Gupta and Golhar [17] has done this kind of work for the case of 100% inspection.

2. Determination of the optimum length of production run for the case of a rectifying inspection.

3. A further research considering both the variables and attribute target mean for the case of a rectifying inspection. This kind of situation appear in the paper, glass, plastic and pharmaceuticals industries. Arcelus and Rahim [3] have developed a model considering single machine with 100% inspection.

4. Usually process variance is an unknown parameter. A research for the determination of the confidence interval for the optimal process setting for the case of an unknown process variance with rectifying inspection is recommended. Recently, a similar kind of work has done by Mihalko and Golhar [24] for the case of 100% inspection.

5. Studying the inspection error effects on the economic selection of target value for the case of rectifying inspection. Chen and Chung [10] has developed a model considering inspection error for the case of Hunter and Kartha [19] where 100% inspection was assumed.

6. All the above work can be extended for the case of two machines with different inspection criteria.

7. Constructing a model for process targeting, production and inventory with reprocessing, recycling and inspection with different criteria for the case of a network of machines by generalizing the ideas of the above works. A small step in this area has been done by Tayi and Ballou [34]. They have developed a model for the case of a serial production systems where reprocessing and inspection are performed after each machine. Their model calcuates the optimal lot size and reprocessing batch size which minimize the total system cost.

Appendix A

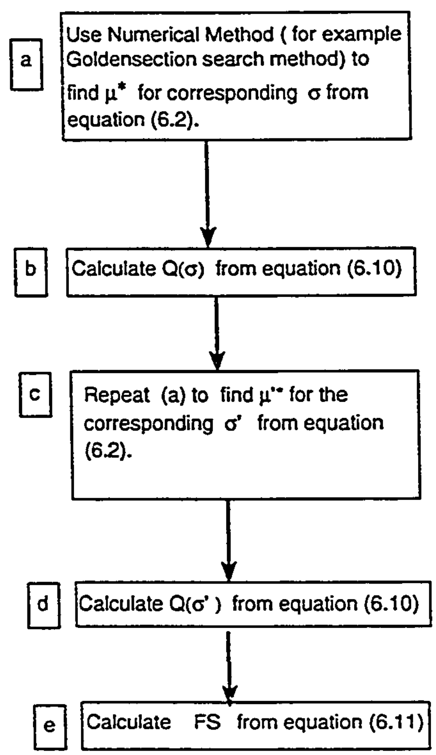

Figure A.1: Flow Chart for computing FS.

Appendix B

```
C===================================================I
C MAIN  PROGRAM
C PROCESS TARGETING PACKAGE
C (FORTRAN PROGRAM OF 10  INDUSTRIAL PROCESS
C TARGETING   MODELS)
C===================================================I
C---------------------------------------------------
C  Comment:1  Length of parameter value NV allows no.
C of iteration to be evaluated from subprogram
C ZBRAC, RTBIS,GOLSEC,HOOK. It could be changed
C according to need.
C---------------------------------------------------
       PARAMETER (NV=250)
       DIMENSION A(NV),B(NV),T(NV),U(NV)
       CHARACTER*1 ANS
```

```
      CHARACTER*64 FILNME
      REAL LL
      COMMON AS,C,S,LL,CO,PP/KHLD/A11,A12,A13,N1,
     + ND1,N2,ND2,S1,S2,RL1,RL2,C1,C2/KHLD1/FX/ARA
     + /BO,BR,BA,G1,G2,RLL,RL,UP,NR,KR /DC1/P/DC3/
     + RCL/DC4/R/BI/N/BIC/K/LOT/NN/MOD/MODEL/EHK/
     + A1,A2,CI/ES/IFMT
C
      WRITE(*,100)
100   FORMAT(/' PROCESS TARGETING PACKAGE'//)
C
      WRITE(*,2)
 2    FORMAT(' WHICH MODEL DO YOU WANT?'//
     +    ' WRITE ===='//
     +    '        1 --- FOR HUNTER AND KARTHA '//
     +    '        2 --- FOR BISGAARD ET. AL '//
     +    '        3 --- FOR CARLSSON '//
     +    '        4 --- FOR GOLHAR '//
     +    '        5 --- FOR SCHMIDT & PFEIFER ' //
     +    '        6 --- FOR BOUCHER & JAFARI'//
     +    '        7 --- FOR PULAK & AL-SULTAN '//
```

```
     +    '          8 --- FOR GOLHAR & POLLACK   '//
     +    '          9 --- FOR ARCELUS AND RAHIM ' //
     +    '          10--- FOR AL-SULTAN  ')
      READ (*,*)MODEL
C
      IF(MODEL.EQ.2)THEN
           WRITE(*,3)
 3         FORMAT('  FIXED MANUFACTURING COST')
           READ (*,*)CO
      ENDIF
      IF(MODEL.EQ.1.OR.MODEL.EQ.3)THEN
           WRITE(*,4)
 4         FORMAT(' SELLING PRICE FOR REJECTED ITEMS')
           READ (*,*)R
      ENDIF
      IF(MODEL.EQ.2)THEN
           WRITE(*,69)
 69        FORMAT(' SELLING PRICE FOR REJECTED ITEMS',
     + '  PER UNIT WEIGHT')
           READ (*,*)R
```

```
      ENDIF
C
      IF(MODEL.EQ.3)THEN
          WRITE(*,83)
 83       FORMAT('  P VALUE ')
          READ (*,*)PP
      ENDIF
C
      IF(MODEL.EQ.1.OR.MODEL.EQ.3)THEN
          WRITE(*,84)
 84       FORMAT(' GIVE AWAY COST PER UNIT WEIGHT')
          READ (*,*)C
      ENDIF
C
      IF(MODEL.EQ.6.OR.MODEL.EQ.7)THEN
          WRITE(*,1)
 1        FORMAT(' SAMPLE SIZE')
          READ (*,*)N
          WRITE(*,11)
 11       FORMAT(' ALLOABLE NO. OF DEFECTIVES')
          READ (*,*)K
```

```
      ENDIF
C
      IF(MODEL.EQ.7)THEN
          WRITE(*,521)
 521      FORMAT(' SELLING PRICE OF 100% SCREENED',
     + '  CAN')
          READ (*,*)A1
          WRITE(*,522)
 522      FORMAT('  SELLING PRICE OF ACCEPTENCE',
     + '  SAMPLING PRODUCT')
          READ(*,*)A2
          WRITE(*,523)
 523      FORMAT('  COST OF INSPACTION PER ITEM FOR',
     + '  REJECTED LOT')
          READ (*,*)CI
          WRITE(*,524)
 524      FORMAT('  LOT SIZE')
          READ (*,*)NN
      ENDIF
C
```

```
      IF(MODEL.LE.6.OR.MODEL.EQ.8)THEN
         WRITE(*,119)
119   FORMAT(' SELLING PRICE OF AN ACCEPTED ITEM')
         READ (*,*)AS
      ENDIF
C
      IF(MODEL.EQ.6)THEN
         WRITE(*,160)
160   FORMAT(' PRICE REDUCTION PER ITEM FROM'
     + ' REJECTED LOT')
         READ (*,*)R
      ENDIF
C
      IF(MODEL.EQ.4.OR.MODEL.EQ.5.OR.
     + MODEL.EQ.7.OR.MODEL.EQ.8)THEN
         WRITE(*,110)
110   FORMAT(' REPROCESSIG COST OF A REJECTED ITEM')
         READ (*,*)R
      ENDIF
C
      IF(MODEL.EQ.2.OR.MODEL.GE.4.AND.MODEL.LE.8)THEN
```

```
      WRITE(*,121)
121  FORMAT(' PER UNIT COST OF FILLING INGREDIENT')
      READ (*,*)C
     ENDIF
C
     IF(MODEL.LE.8)THEN
      WRITE(*,122)
122  FORMAT(' STANDARD DEVIATION OF THE PROCESS   ')
      READ (*,*)S
      WRITE(*,123)
123  FORMAT(' LOWER SPECIFICATION LIMIT')
      READ (*,*)LL
     ENDIF
C
     IF(MODEL.EQ.9)THEN
      WRITE(*,919)
919  FORMAT(' INCOME EARNED PER ACCEPTED  ITEM')
      READ (*,*)BA
      WRITE(*,920)
920  FORMAT(' INCOME EARNED PER REJECTED  ITEM')
```

```
      READ (*,*)BR
      WRITE(*,921)
921   FORMAT(' FIXED COMPONENT COST OF PRODUCING'
     + ' AN ITEM')
      READ (*,*)BO
      WRITE(*,922)
922   FORMAT(' COST TO MAINTAIN VARIABLE QUALITY'
     + ' CHARACTERISTIC PER ITEM')
      READ (*,*)G1
      WRITE(*,923)
923   FORMAT(' COST TO MAINTAIN ATTRIBUTE QUALITY'
     + ' CHARACTERISTIC PER ITEM')
      READ (*,*)G2
      WRITE(*,924)
924   FORMAT(' SAMPLE SIZE')
      READ (*,*)NR
      WRITE(*,925)
925   FORMAT(' ALLOABLE NO. OF DEFECTIVES')
      READ (*,*)KR
      WRITE(*,926)
926   FORMAT(' ACCEPTABLE UPPER LIMIT FOR ATTRIBUTE'
```

```
     + ' CHARACTERISTIC')
        READ (*,*)RL
        WRITE(*,927)
927   FORMAT(' ACCEPTABLE UPPER LIMIT FOR VARIABLE'
     + ' CHARACTERISTIC')
        READ (*,*)UP
        WRITE(*,928)
928   FORMAT(' ACCEPTABLE LOWER LIMIT FOR VARIABLE'
     + ' CHARACTERISTIC')
        READ (*,*)RLL
      ENDIF
C
      IF(MODEL.EQ.10)THEN
        WRITE(*,19)
19    FORMAT(' SELLING PRICE PER ITEM FOR ITEMS'
     + ' REJECTED AFTER MACHINE 1')
        READ (*,*)A11
        WRITE(*,20)
20    FORMAT(' SELLING PRICE PER ITEM FOR ITEMS'
     + ' REJECTED AFTER MACHINE 2')
```

```
      READ (*,*)A12
      WRITE(*,21)
21    FORMAT(' SELLING PRICE PER ITEM FOR'
     + ' ACCEPTED ITEMS')
      READ (*,*)A13
      WRITE(*,22)
22    FORMAT(' SAMPLE SIZE FOR SAMPLING'
     + ' AFTER  MACHINE 1')
      READ (*,*)N1
      WRITE(*,23)
23    FORMAT(' SAMPLE SIZE FOR SAMPLING'
     + ' AFTER  MACHINE 2')
      READ (*,*)N2
      WRITE(*,24)
24    FORMAT(' ALLOWABLE NO. OF NONCONFORMING'
     + ' ITEMS FOUND IN SAMPLE OF SIZE N1')
      READ (*,*)ND1
      WRITE(*,25)
25    FORMAT(' ALLOWABLE NO. OF NONCONFORMING'
     + '  ITEMS FOUND IN SAMPLE OF SIZE N2')
      READ (*,*)ND2
```

```
      WRITE(*,28)
28    FORMAT(' PROCESS VARRIENCE FOR MACHINE 1')
      READ (*,*)S1
      WRITE(*,29)
29    FORMAT(' PROCESS VARRIENCE FOR MACHINE 2')
      READ (*,*)S2
      WRITE(*,30)
30    FORMAT(' LOWER SPECIFICATION LIMIT FOR THE'
     + ' 1ST PRODUCT ')
      READ (*,*)RL1
      WRITE(*,31)
31    FORMAT(' LOWER SPECIFICATION LIMIT FOR THE'
     + ' 2ND PRODUCT ')
      READ (*,*)RL2
      WRITE(*,32)
32    FORMAT(' COST OF PROCESSING PER UNIT OF THE'
     + ' 1ST PRODUCT  ')
      READ (*,*)C1
      WRITE(*,33)
33    FORMAT(' COST OF PROCESSING PER UNIT OF THE '
```

```
     + ' 2ND PRODUCT  ')
        READ (*,*)C2
       ENDIF
C
       WRITE(*,670)
670    FORMAT(/' DO YOU WANT OUTPUT SENT TO THE '
     + ' SCREEN OR TO A FILE?'/
     + '        WRITE (S) FOR SCREEN,(F) FOR FILE')
       READ(*,666)ANS
C
       IF(ANS.EQ.'F'.OR.ANS.EQ.'f')THEN
            WRITE(*,621)
621       FORMAT(/' WRITE UNIT NO. OF OUTPUT FILE')
            READ(*,*)IFMT
            WRITE(*,671)
671       FORMAT(/' ENTER NAME OF YOUR OUTPUT FILE  '
     +    ' (NAME LENGTH WITHIN 64 CHARACTER):')
            READ(*,667)FILNME
            OPEN(UNIT=IFMT,FILE=FILNME)
       ENDIF
C
```

```
      IF(MODEL.EQ.1.OR.MODEL.EQ.3)THEN
         XI=C*S/R
         X0=X00(XI)
      ENDIF
C
      IF(MODEL.EQ.2)THEN
         XI=(C/R)/((AS/R)-LL)*S
         X0=X00(XI)
      ENDIF
C
      IF(MODEL.EQ.4)THEN
        XI=R/(C*S)
        X0=0.712+0.471*ALOG(XI)
      ENDIF
C
      IF(MODEL.EQ.5)THEN
        XI=(AS+R-C*LL)/(C*S)
        X0=EXP(0.00081/XI+0.446-0.203*XI-0.029*
     +  XI*XI+0.00104*Q*Q*Q)-1.57
         WRITE(*,50)
```

```
50        FORMAT(//' FOR MEAN OF THE PROCESS LEVEL')
      ENDIF
C
      IF(MODEL.EQ.6)THEN
            XI=C*S/R
                IF(K.EQ.0)THEN
                    XI=XI/N
                    XO=XOO(XI)
                ENDIF
                IF(K.EQ.1)THEN
                    XI=(XI/N)*17
                    XO=XOO(XI)
                ENDIF
                IF(K.GE.2)THEN
                    XI=(XI/N)*17*EXP((1.-K/N)*(1.-K/N))
                    XO=XOO(XI)
                ENDIF
      ENDIF
C
      IF(MODEL.EQ.7)THEN
            XI=(C*S/R)*(A1-A2+1)
```

```
      IF(K.EQ.0)THEN
          XI=XI/N
          XO=XOO(XI)
      ENDIF
      IF(K.EQ.1)THEN
         XI=(XI/N)*17
        XO=XOO(XI)
      ENDIF
      IF(K.GE.2)THEN
         XI=(XI/N)*17*EXP((1.-K/N)*(1.-K/N))
         XO=XOO(XI)
      ENDIF
   ENDIF
C
   IF(MODEL.EQ.8)THEN
         GP=R/(C*S)
         Y22=-(0.746*SQRT(GP))
         Y11=-(2*Y22)
   ENDIF
C
```

```
      IF(MODEL.EQ.9)THEN
            Y11=0.5*(UP-RLL)
            Y22=RL
       ENDIF
C
       IF (MODEL.EQ.10)THEN
             XI=(C2*S2)/(A13-A12)
                 IF(ND2.EQ.0)THEN
                      XI=XI/N2
                     X0=X00(XI)
                 ENDIF
                IF(ND2.EQ.1)THEN
                     XI=(XI/N2)*17
                     X0=X00(XI)
                ENDIF
                IF(ND2.GE.2)THEN
                    XI=(XI/N2)*17*EXP((1.-ND2/N2)*
     +                   (1.-ND2/N2))
                    X0=X00(XI)
                 ENDIF
        ENDIF
```

123

```
C
      IF(MODEL.LE.7.OR.MODEL.GE.10)THEN
        IF(MODEL.EQ.10)THEN
             LL=RL2
             S=S2
             WRITE(*,*)
             WRITE(*,*)' FOR 2ND MACHINE: '
        ENDIF
       IF(MODEL.EQ.5)X0=-X0
       UM=LL+S*X0
       WRITE(*,*)
       WRITE(*,907)UM
 907  FORMAT(' SUGGESTED STARTING MEAN SETTING'
     +   ' IS:',F18.9)
        WRITE(*,*)
        WRITE(*,908)
 908  FORMAT(/' DO YOU WANT TO CONTINUE WITH'
     +  ' SUGGESTED STARTING VALUE?'/
     +   '      WRITE (Y) FOR YES AND (N) FOR NO')
        READ(*,666)ANS
```

123

```
      IF(ANS.EQ.'N'.OR.ANS.EQ.'n')THEN
           WRITE(*,*)
           WRITE(*,909)
909     FORMAT('  WRITE YOUR STARTING VALUE')
           READ(*,*)UM
           X0=(UM-LL)/S
          IF(MODEL.EQ.5)X0=-X0
      ENDIF
      IF(MODEL.LE.7)THEN
         CALL GOLSEC(NV,X0,A,B,T,U,LL,S,FILNME)
      ENDIF
      IF(MODEL.EQ.10)THEN
          X1=X0
         CALL  ZBRAC(X1,X2,NV)
          X11=X1
          X24=X2
          XACC=0.00001
          D=RTBIS(X11,X24,XACC,NV)
          PYY=1-PNORM(D)
          XXX=CUMB(N2,ND2,PYY)
          GMU2=RL2+D*S2
```

```
      FX=C2*GMU2+A11-A12+(A12-A13)*XXX
      WRITE(IFMT,801)
801   FORMAT(/' PROCESS TARGETING PACKAGE'//)
      WRITE(IFMT,*)
      WRITE(IFMT,*)
      WRITE(IFMT,*)
      WRITE(IFMT,18)MODEL
 18   FORMAT(' MODEL NO. =',I3)
      WRITE(IFMT,*)
C
      IF(FX.GE.0)THEN
         WRITE(IFMT,109)
 109     FORMAT(' USE MACHINE ONE ONLY     ')
         STOP
         ELSE
C
         WRITE(IFMT,*)
         WRITE(IFMT,108)GMU2
 108     FORMAT(' PROCESS MEAN SETTING FOR MACHINE 2 '
     +   ,F18.9)
      ENDIF
```

```
              MODEL=11
          ENDIF
C
        IF (MODEL.EQ.11)THEN
           XI=(-C1*S1)/FX
              IF(ND1.EQ.0)THEN
                  XI=XI/N1
                  X0=X00(XI)
              ENDIF
              IF(ND1.EQ.1)THEN
                 XI=(XI/N1)*17
                 X0=X00(XI)
              ENDIF
              IF(ND1.GE.2)THEN
                 XI=(XI/N1)*17*EXP((1.-ND1/N1)*(1.-ND1/N1))
                 X0=X00(XI)
                 LL=RL1
                S=S1
                WRITE(*,*)
                WRITE(*,*)' FOR 1ST MACHINE: '
```

```
          ENDIF
        IF(MODEL.EQ.5)X0=-X0
        UM=LL+S*X0
        WRITE(*,*)
        WRITE(*,917)UM
917     FORMAT(' SUGGESTED STARTING MEAN SETTING'
     +  '  IS:',F18.9)
        WRITE(*,*)
        WRITE(*,918)
918     FORMAT(/' DO YOU WANT TO CONTINUE WITH'
     +  ' SUGGESTED STARTING VALUE?'/
     + '    WRITE (Y) FOR YES AND (N) FOR NO')
        READ(*,666)ANS
              IF(ANS.EQ.'N'.OR.ANS.EQ.'n')THEN
                 WRITE(*,*)
                 WRITE(*,979)
979           FORMAT('  WRITE YOUR STARTING VALUE')
                 READ(*,*)UM
                 X0=(UM-LL)/S
                IF(MODEL.EQ.5)X0=-X0
            ENDIF
```

```
      X1=X0
      CALL  ZBRAC(X1,X2,NV)
      X11=X1
      X24=X2
      XACC=0.00001
      D=RTBIS(X11,X24,XACC,NV)
      P=1-PNORM(D)
      X11=CUMB(N1,ND1,P)
      GMU1=RL1+D*S1
      WRITE(IFMT,*)
      WRITE(IFMT,*)
      WRITE(IFMT,131)GMU1
131   FORMAT(' PROCESS MEAN SETTING FOR MACHINE 1 '
     + ,F18.9)
      WRITE(IFMT,*)
      WRITE(IFMT,*)
      FXT= A11*(1.- X11)+A12*X11*(1.-XXX)+A13*X11*XXX
     +     -GMU1*C1-X11*GMU2*C2
      WRITE(IFMT,129)FXT
129   FORMAT(' MAXIMUM PROFIT PER ITEM =    ',F12.6)
```

```
      ENDIF
     ENDIF
C
     IF(MODEL.EQ.8.OR.MODEL.EQ.9)THEN
        IF(MODEL.EQ.8)THEN
            UM=LL-S*Y22
            UPP=UM+Y11*S
            WRITE(*,*)
            WRITE(*,96) UM
 96          FORMAT(' SUGGESTED STARTING MEAN '
   +     '   SETTING IS',F18.9)
            WRITE(*,97) UPP
 97          FORMAT(' SUGGESTED STARTING '
   +     ' UPPER LIMIT IS',F18.9)
           WRITE(*,*)
           WRITE(*,988)
 988        FORMAT(' DO YOU WANT TO CONTINUE WITH '
   + '    SUGGESTED STARTING VALUES?'/
   +   '   WRITE (Y) FOR YES, (N) FOR NO')
           READ(*,666)ANS
              IF(ANS.EQ.'N'.OR.ANS.EQ.'n')THEN
```

```
                 WRITE(*,*)
                 WRITE(*,990)
990              FORMAT(' WRITE YOUR STARTING VALUE FOR '
    +       '     MEAN SETTING')
                 READ(*,*)UM
                 Y22=(LL-UM)/S
                 WRITE(*,989)
989              FORMAT(' WRITE YOUR STARTING VALUE FOR'
    +       '     UPPER LIMIT')
                 WRITE(*,*)
                 READ(*,*)UPP
                 Y11=(UPP-UM)/S
            ENDIF
      ENDIF
C
      IF(MODEL.EQ.9)THEN
            WRITE(*,*)
            UM=UP-Y11
            WRITE(*,196) UM
196         FORMAT(' SUGGESTED STARTING VALUE '
```

```
     +    '   FOR VARRIABLE CHARACTERISTICS IS',F18.9)
              WRITE(*,197) Y22
197        FORMAT(' SUGGESTED STARTING VALUE'
     +     ' FOR ATTRIBUTE CHARACTERISTICS IS',F18.9)
              WRITE(*,958)
958        FORMAT(' DO YOU WANT TO CONTINUE WITH '
     + '  SUGGESTED STARTING VALUES?'/
     +    '  WRITE (Y) FOR YES, (N) FOR NO')
              READ(*,666)ANS
                 IF(ANS.EQ.'N'.OR.ANS.EQ.'n')THEN
                     WRITE(*,*)
                     WRITE(*,950)
950                FORMAT(' WRITE YOUR STARTING VALUE FOR '
     +      '         VARRIABLE CHARACTERISTICS ')
                     READ(*,*)UM
                     Y11=UP-UM
                     WRITE(*,959)
959                FORMAT(' WRITE YOUR STARTING VALUE FOR'
     +              '   ATTRIBUTE CHARACTERISTICS ')
                     WRITE(*,*)
                    READ(*,*)Y22
```

123

```
                 ENDIF
         ENDIF
C
        CALL HOOK(Y11,Y22,H1,H2,VALHOK,NV,FILNME)
        ENDIF
C
666      FORMAT(A1)
667      FORMAT(A64)
         STOP
         END
C=====================================================I
C SUB PROGRAM:1
CINITIAL STARTING SOLUTION
C(THIS SUBPROGRAM IS BASED ON NELSON[1978])
C
C=====================================================I
        FUNCTION X00(XI)
        XIN=4+ALOG10(XI)
        X00=4.07-0.545*XIN-0.054*XIN*XIN-10**
      + (-2.73+0.712*XIN)
```

```
      RETURN
      END
C==================================================I
CSUB PROGRAM:2
CROOT BRACKETING
CWITH THE INITIAL GAUSSED VALUE X1 THIS SUBROUTINE
CEXPANDS GEOME-TRICALLY UNTIL A ROOT IS BRACKETED &
CBY THE RETURNED VALUES X1 & X2 .THIS CODE IS
CFROM Press et. al [1987]
C==================================================I
      SUBROUTINE  ZBRAC(X1,X2,NV)
      PARAMETER(FACTOR=1.6)
      X2=X1+1.5
C-------------------------------------------------
C Comment 2: Possible underflow has been avoided if
Ceither X1 or and X2 equals 0.This kind of underflow
Cnot necessarily occur from that particular subprogram
Cbut also from other subprogram.This precaution has
Calso been applied taken in other subprograms. If X1
Cor X2 or both equals 0, then its value could be
Cassumed according to user's need and machine
```

```
Cprecision.
C
C-----------------------------------------------------
      IF(X1.EQ.0.)X1=.0001
      IF(X2.EQ.0.)X2=.0001
      F1=F(X1)
      F2=F(X2)
      DO 11 J=1,NV
 IF(F1*F2.LT.0.)RETURN
 IF(ABS(F1).LT.ABS(F2))THEN
   X1=X1+FACTOR*(X1-X2)
   F1=F(X1)
        ELSE
           X2=X2+FACTOR*(X2-X1)
           F2=F(X2)
          ENDIF
 C-----------------------------------------------------
 C Comment 3: See Comment 2.
 C
 C-----------------------------------------------------
```

123

```
 IF(X1.EQ.0.)X1=.0001
 IF(X2.EQ.0.)X2=.0001
C--------------------------------------------------
C Comment 4: Value of DVI & VID are changable.
C
C--------------------------------------------------
DIV=ABS(10)
IF(X1.GE.DIV.OR.X2.GE.DIV)THEN
     WRITE(*,119)
119      FORMAT(' TRY WITH OTHER INITIAL VALUE'
     +    ' OR INCREASE DIV VALUE FROM SUBPROGRAM-2')
    STOP
ENDIF
VID=ABS(1.E+30)
IF(F(X1).GE.VID.OR.F(X2).GE.VID)THEN
    WRITE(*,120)
120      FORMAT(' TRY WITH OTHER INITIAL VALUE'
     +    ' OR INCREASE VID VALUE FROM SUBPROGRAM-2')
    STOP
ENDIF
 11   CONTINUE
```

123

```
      PAUSE' NO. OF ALLOWABLE ITERATION EXCEEDS(NV).'
      PAUSE' TRY WITH OTHER STARTING SOLUTION'
      RETURN
      END
C=====================================================I
CSUB PROGRAM:3
CROOT FINDING
CUSING BISECTION METHOD(See Bazaraa et. al [1993]
Cfor details) THIS SUBROUTINE FINDS THE ROOT OF A
CFUNCTION WHICH LIES BETWEEN X1 &X2.THE ROOT WILL
CBE REFINED UNTIL ITS ACCURACY Abs(XACC).XACC CAN
CBE CHANGED FROM MAIN PROGRAM ACCORDING TO USER'S
CNEED AND MACHINE PRECISION.THIS  CODE IS FROM
CPress et. al [1987].
C=====================================================I
      FUNCTION RTBIS(X1,X2,XACC,NV)
      FMID=F(X2)
      FF=F(X1)
      IF(FF.LT.0.)THEN
  RTBIS=X1
```

```
DX=X2-X1
     ELSE
RTBIS=X2
DX=X1-X2
     ENDIF
     DO 11 J=1,NV
DX=DX*.5
XMID=RTBIS+DX
FMID=F(XMID)
IF(FMID.LE.0.)RTBIS=XMID
IF(ABS(DX).LT.XACC.OR.FMID.EQ.0.)RETURN
11   CONTINUE
     END
C====================================================I
CSUB PROGRAM:4
CEVALUATION OF MODEL
C1-7 MODEL'S PROFIT FUNCTION IS EVALUATED HERE.
CDIFERENTIAL EQUATION SOF MODEL 10 IS EVALUATED HERE.
C
       REAL FUNCTION F(XO)
       PARAMETER(TINY=-203.0)
```

```
      COMMON AS,C,S,LL,C0,PP/KHLD/A11,A12,A13,N1,ND1
     +,N2,ND2,S1,S2,RL1,RL2,C1,C2/KHLD1/FX/NRC/RCL22/
     + NT/T/DC4/R/DC1/P/BI/N /BIC/K/DC3/RCL/LOT/NN/
     + MOD/MODEL/SCHMDT/US/EHK/A1,A2,CI
      REAL MU,LL
      P=1.-PNORM(X0)
      NU=N
      KU=K
      PU=P
      IF(MODEL.EQ.1)GO TO 100
      IF(MODEL.EQ.2)GO TO 200
      IF(MODEL.EQ.3)GO TO 300
      IF(MODEL.EQ.4)GO TO 400
      IF(MODEL.EQ.5)GO TO 500
      IF(MODEL.EQ.6)GO TO 600
      IF(MODEL.EQ.7)GO TO 700
      IF(MODEL.EQ.10)GO TO 1000
      IF(MODEL.EQ.11)GO TO 1100
100   MU=LL+X0*S
      F=-(PNORM(X0)*(AS-C*MU+C*LL-R)+R-C*S*ORDN(X0))
```

123

```
      RETURN
 200  MU=LL+X0*S
      F=-((1-PNORM(X0))*(MU*R-AS)-(R*S*ORDN(X0))
     +  -C*MU+AS-CO)
      RETURN
 300  F=-(AS-(AS-R)*(1-PNORM(X0))+(PP*C*S)*(X0*
     + (1-PNORM(X0))-ORDN(X0)-X0/PP))
      RETURN
 400  MU=LL+X0*S
      F=-(AS-C*MU-(1./PNORM(X0))*(R*(1.-PNORM(X0))+
     + C*S*ORDN(X0)))
      RETURN
 500  US=(AS+R)/C
      UM=LL-S*X0
      T1=(US-UM)/S
      F=-((AS+R-C*UM)*(PNORM(T1)-PNORM(X0))-
     + R+C*S*(ORDN(T1)-ORDN(X0)))
      RETURN
 600  MU=LL+X0*S
      F=-(-(R*(1.-CUMB(NU,KU,PU)))+AS-C*MU)
      RETURN
```

```
 700    MU=LL+X0*S
         DKK=NN

         DK=NU

          RCU=(1-CUMB(NU,NP,PU))

          IF(RCU.EQ.0)RCU=1.E-20
          AC=(RCLT(NU,NP,PU,R)
/R)/RCU

          AB=PU*(DKK-NU)

          RC=R*(AC+AB)/DKK

         PY=DK/DKK
         F=-((A2-A1)*CUMB(NU,KU,PU)-((RC+(1-PY)*CI)*

     +    (1.-CUMB(NU,KU,PU)))+A1-C*MU)

        RETURN

  1000  P2=P

           IF(P2.EQ.0)P2=1.E-20

         NU2=N2

         NP2=ND2

         Q2=1-P2

          IF(Q2.EQ.0)Q2=1.E-20

         RHS=C2/(A12-A13)

         NP21=NU2-1

         NP22=NU2-1-ND2
```

```
      BICON1=(FACTLN(NP21)-FACTLN(NP22)-FACTLN(NP2)
     + +NP2*ALOG(P2)
     + +NP22*ALOG(Q2))
      IF(BICON1.LE.-175)THEN
      T=0.
      ELSE
      T=NU2*EXP(BICON1)
      ENDIF
      HLS=-ORDN(X0)*(1/S2)*T
      F=(HLS-RHS)
      RETURN
 1100 RHS=C1/FX
      PU1=1.-PNORM(X0)
       IF(PU1.EQ.0)PU1=1.E-20
      NU1=N1
      NP1=ND1
      PQ1=1-PU1
        IF(PQ1.EQ.0)PQ1=1.E-50
      NP11=NU1-1
      NP12=NU1-1-ND1
      BICON1=(FACTLN(NP11)-FACTLN(NP12)-FACTLN(NP1)
```

```
     + +NP1*ALOG(PU1)
     + +NP12*ALOG(PQ1))
       IF(BICON1.LE.-55)THEN
       T=0.
       ELSE
       T=N1*EXP(BICON1)
       ENDIF
       HLS=-ORDN(X0)*(1/S1)*T
       F=(HLS-RHS)
       RETURN
       END
C==================================================I
CSUB PROGRAM:5
CEVALUATION OF MODEL
C8 & 9 MODEL'S PROFIT FUNCTION HAS EVALUATED HERE.
C
      SUBROUTINE VAL(XK1,XK2,FXT)
      COMMON AS,C,S,LL,C0,PP/KHLD/A11,A12,A13,N1,ND1
     + ,N2,ND2,S1,S2,RL1,RL2,C1,C2/ARA/B0,BR,BA,G1,G2,
     + RLL,RL,UP,NR,KR/DC4/R/MOD/MODEL
```

```
      REAL LL
      DD=XK1
      HH=XK2
      IF (MODEL.EQ.8)GO TO 800
      IF (MODEL.EQ.9)GO TO 900
  800 GMU=LL-S*HH
      FXT1=R+C*S*(ORDN(HH)-ORDN(DD))
      FXT2=PNORM(DD)-PNORM(HH)
      AFF=FXT1/FXT2
      FXT=-(AS-C*GMU+R-AFF)
      RETURN
  900 BAR=BA-BR
      BRO=BR-BO
      RL1=RL
      RL1=RL1+1
      GMU=UP-XK1
      PU1=1.-(PNORM(XK1)-PNORM(RLL-GMU))*
     +      GAMMQ(RL1,XK2)
      NU=NR
      NP=KR
      FXT=-(BRO+BAR*CUMB(NU,NP,PU1)-G1*GMU-G2*EXP(RL-XK2))
```

```
RETURN
END
C====================================================I
CSUB PROGRAM:6
CONE DIMENSIONAL OPTIMIZATION
CWITH THE INITIAL STARTING VALUE THIS SUBROUTINE WILL
CBRACKET THE MINIMUM AND THEN BY GOLDEN SECTION SEARCH
C(See Bazaraa et. all [1993] )IT WILL REFINE THE
CMINIMUM WITH THE ACCURACY 'L'. THE VALUE OF L
CCAN BE CHANGED ACCORDING TO USER'S NEED AND
CMACHINE PRECISION MODEL 1 THROUGH 7 ARE
COPTIMIZED HERE.
C
SUBROUTINE GOLSEC(N,X0,A,B,T,U,LL,S,FILNME)
COMMON/KHLD1/FX/MOD/MODEL/SCHMDT/US/EHK/A1,
     + A2,CI/ES/IFMT
 CHARACTER*64 FILNME
         REAL L,LL
DIMENSION A(N), B(N), T(N),U(N)
IF(IFMT.GT.0)THEN
```

```
   OPEN(UNIT=IFMT,FILE=FILNME)
ENDIF
        L=.0000001
        DA=.5
DO 40 K=1,N
X1=X0-DA
X2=X0+DA
C---------------------------------------------------
C Comment 5: See Comment 2. DIV & VID value are
C     changable.
C---------------------------------------------------
DIV=ABS(10)
IF(X1.GE.DIV.OR.X2.GE.DIV)THEN
    WRITE(*,119)
119     FORMAT(' TRY WITH OTHER INITIAL VALUE'
     +    ' OR INCREASE DIV VALUE FROM SUBPROGRAM-6')
   STOP
ENDIF
         IF(X1.EQ.0.D0)X1=.0001
         IF(X2.EQ.0.D0)X2=.0001
         IF(N.GT.250) THEN
```

```
  PAUSE' NO. OF ITERATION EXCEEDS 250(NV)'
  PAUSE' TRY WITH OTHER STARTING VALUE'
  STOP
ENDIF
VID=ABS(1.E+30)
IF(F(X1).GE.VID.OR.F(X2).GE.VID)THEN
  WRITE(*,120)
120   FORMAT(' TRY WITH OTHER INITIAL VALUE'
     +    ' OR INCREASE VID VALUE FROM SUBPROGRAM-6')
  STOP
ENDIF
        IF(F(X1).GE.F(X0).AND.F(X0).GE.F(X2))GOTO 10
IF(F(X1).LE.F(X0).AND.F(X0).LE.F(X2))GOTO 20
IF(F(X1).GE.F(X0).AND.F(X0).LE.F(X2))GOTO 30
  10 X0=X2
GOTO 40
  20 X0=X1
GOTO 40
  30 A(1)=X1
B(1)=X2
```

```
GOTO 1
 40 CONTINUE
 1 ALPHA= 0.618
T(1)=A(1)+(1-ALPHA)*(B(1)-A(1))
U(1)=A(1)+ALPHA*(B(1)-A(1))
C
DO 4 K=1,N
J=K
IF((B(K)-A(K)).LT.L) GOTO 100
C-----------------------------------------------------
C Comment 6: See Comment 2.
C
C-----------------------------------------------------
IF(T(K).EQ.0.D0) T(K)=.0001
IF(U(K).EQ.0.D0) U(K)=.0001
IF(F(T(K)).GT.F(U(K))) GOTO 2
IF(F(T(K)).LE.F(U(K))) GOTO 3
C
 2 A(K+1)=T(K)
B(K+1)=B(K)
T(K+1)=U(K)
```

```
U(K+1)=A(K+1)+ALPHA*(B(K+1)-A(K+1))
GOTO 4
  3 A(K+1)=A(K)
B(K+1)=U(K)
U(K+1)=T(K)
T(K+1)=A(K+1)+(1-ALPHA)*(B(K+1)-A(K+1))
  4 CONTINUE
 100 PRINT*
C
          WRITE(IFMT,801)
801    FORMAT(/' PROCESS TARGETING PACKAGE'//)
  WRITE(IFMT,*)
  WRITE(IFMT,*)
  WRITE(IFMT,11)MODEL
  11   FORMAT(' MODEL NO. =  ',I3)
          WRITE(IFMT,*)
  WRITE(IFMT,*)
 C
  DO 6 I=1,J
  F1=-F(A(I))
```

```
C
             IF (MODEL.EQ.5) THEN
           A(I)=-A(I)
        ENDIF
C
GMU=LL+A(I)*S
  6 CONTINUE
C
WRITE(IFMT,25)A(j)
  25 FORMAT(' STANDARDIZED EXCESS LEVEL =',F10.6)
WRITE(IFMT,*)
WRITE(IFMT,15)F1
  15 FORMAT(' PROFIT PER ITEM  ='F10.6)
WRITE(IFMT,*)
WRITE(IFMT,48)GMU
 48 FORMAT(' MEAN OF THE PROCESS SETTING=',F13.6)
C
        IF(MODEL.EQ.5)THEN
           WRITE(IFMT,8)US
  8       FORMAT(' UPPER CONTROL LIMIT=',F18.9)
        ENDIF
```

```
C
RETURN
END
C==================================================I
CSUB PROGRAM:7
CTWO DIMENSIONAL OPTIMIZATION
CWITH THE INITIAL STARTING VALUES THIS SUBROUTINE
CWILL SEARCH FOR THE MINIMUM BY HOOK AND JEEVES
CALGORITHM(See Bazaraa et. al [1993] )IT WILL
CREFINE THE MINIMUM WITH THE ACCURA "STEP".
CTHE VALUE OF STEP CAN BE CHANGED ACCORDING TO
CUSER'S NEED AND MACHINE PRECISION. MODEL 8 & 9
CARE OPTIMIZED HERE.
C
SUBROUTINE HOOK(Y11,Y22,H1,H2,VALHOK,NV
     + ,FILNME)
REAL XO(2),XM(2),XX(2),XT(2),LL
COMMON AS,C,S,LL,CO,PP/DC4/R/ARA/BO,BR,BA,
     + G1,G2,RLL,RL,UP,
     + NR,KR/MOD/MODEL/ES/IFMT
```

```
C
CHARACTER*64 FILNME
IF(IFMT.GT.0)THEN
  OPEN(UNIT=IFMT,FILE=FILNME)
ENDIF
XO(1)=Y11
XO(2)=Y22
XM(1)=XO(1)
XM(2)=XO(2)
XK1=XM(1)
XK2=XM(2)
C---------------------------------------------------
C Comment 7: See Comment 2.
C
C---------------------------------------------------
        IF(XK1.EQ.0.D0)XK1=.0005
        IF(XK2.EQ.0.D0)XK2=.0005
STEP=.1
CALL VAL(XK1,XK2,FXB)
NN=2
        ICOUNT=0
```

```
 21 KK=0
ICOUNT=ICOUNT+1
DO 11 II=1,NN
        TEMP=XM(II)
        XM(II)=XM(II)+STEP
        XK1=XM(1)
        XK2=XM(2)
C--------------------------------------------------
C Comment 8: See Comment 2.
C
C--------------------------------------------------
        IF(XK1.EQ.0.D0)XK1=.0005
        IF(XK2.EQ.0.D0)XK2=.0005
 CALL VAL(XK1,XK2,FXE)
 IF(FXE.LT.FXB)GO TO 12
 XM(II)=TEMP
 XM(II)=XM(II)-STEP
 XK1=XM(1)
 XK2=XM(2)
 C--------------------------------------------------
```

```
C Comment 9: See Comment 2.
C
C------------------------------------------------------
        IF(XK1.EQ.0.D0)XK1=.0005
        IF(XK2.EQ.0.D0)XK2=.0005
CALL VAL(XK1,XK2,FXE)
IF (FXE.LT.FXB)GO TO 12
XM(II)=TEMP
GO TO 11
 12 FXB=FXE
KK=1
 11 CONTINUE
IF(KK.EQ.0)GO TO 18
DO 16 JJ=1,NN
 16     XX(JJ)=XM(JJ)
DO 13 JJ=1,NN
 13     XT(JJ)=2.*XX(JJ)-XO(JJ)
XK1=XT(1)
XK2=XT(2)
C------------------------------------------------------
C Comment 10: See Comment 2.DVI & VDI IS CHANGABLE
```

```
C
C-------------------------------------------------------
        IF(XK1.EQ.0.D0)XK1=.0005
        IF(XK2.EQ.0.D0)XK2=.0005
CALL VAL(XK1,XK2,FXT)
DO 14 MM=1,NN
 14 XO(MM)=XX(MM)
C
DIV=ABS(20)
IF(XK1.GE.DIV.OR.XK2.GE.DIV)THEN
WRITE(*,119)
119 FORMAT(' TRY WITH OTHER INITIAL VALUE'
     +    ' OR INCREASE DIV VALUE FROM SUBPROGRAM-7')
STOP
ENDIF
C
VID=ABS(1.E+30)
IF(FXB.GE.VID)THEN
WRITE(*,120)
120 FORMAT(' TRY WITH OTHER INITIAL VALUE'
```

```
     +    ' OR INCREASE VID VALUE FROM SUBPROGRAM-7')
      STOP
      ENDIF
C
      IF(FXT.LE.FXB) GO TO 15
      GO TO 21
 15   FXB=FXT
      DO 17 II=1,NN
 17   XM(II)=XT(II)
      GO TO 21
 18   IF((STEP.LT..000001.OR.ICOUNT.GT.NV)) GO TO 19
      STEP=STEP/2.
      GO TO 21
 19   CONTINUE
      H1=XK1
      H2=XK2
      VALHOK=-FXB
C
         WRITE(IFMT,801)
 801     FORMAT(/' PROCESS TARGETING PACKAGE'//)
         WRITE(IFMT,*)
```

```
      WRITE(IFMT,28)MODEL
28    FORMAT(' MODEL NO.=',I3)
      WRITE(IFMT,*)
      WRITE(IFMT,*)
C
      IF(MODEL.EQ.8)THEN
           GMU=LL-S*H2
           UP=GMU+S*H1
      EXC=AS-C*LL-VALHOK
      WRITE(IFMT,*)
      WRITE(IFMT,25)GMU
 25       FORMAT(' MEAN OF THE PROCESS SETTING =', F10.6)
      WRITE(IFMT,*)
      WRITE(IFMT,125)UP
 125      FORMAT(' UPPER LIMIT =', F10.6)
      WRITE(IFMT,*)
      WRITE(IFMT,225)VALHOK
 225      FORMAT(' PROFIT PER ITEM =', F10.6)
      WRITE(IFMT,*)
      WRITE(IFMT,325)EXC
```

```
 325      FORMAT(' MINIMUM  EXCESS COST =',F10.6)
ENDIF
C
IF(MODEL.EQ.9)THEN
    GMU1=UP-H1
    RLAMDA=H2
    WRITE(IFMT,75)GMU1
 75     FORMAT(' PROCESS SETTING FOR VARRIABLE = '
     + ,   F10.6)
    WRITE(IFMT,*)
    WRITE(IFMT,175)RLAMDA
 175     FORMAT(' PROCESS SETTING FOR ATTRIBUTE = '
     + ,   F10.6)
    WRITE(IFMT,*)
    WRITE(IFMT,275)VALHOK
 275     FORMAT(' PROFIT PER ITEM = ',F10.6)
    WRITE(IFMT,*)
 ENDIF
   9 CONTINUE
 C
 RETURN
```

```
END
C==================================================I
CSUB PROGRAM:8
CNORMAL DENSITY FUNCTION
CTHIS SUBPROGRAM CALCULATE NORMAL DENSITY FUNCTION
C
FUNCTION ORDN(Z)
DATA SQT2PI/2.506628274631D0/
Y=.5*Z*Z
C--------------------------------------------------
C Comment 11: If Z gt. 10 then ORDN negligibly
Csmall. It can be changed according to user's
Cchoice and depending on machine precission .
C
C--------------------------------------------------
IF (Z.GT.10)THEN
    ORDN=1.E-20
  ELSE
    ORDN=EXP(-Y)/SQT2PI
 ENDIF
```

```
RETURN
END
C=================================================I
CSUB PROGRAM:9
CCUMULATIVE  NORMAL DENSITY FUNCTION
CTHIS SUBPROGRAM CALCULATE CUMULATIVE NORMAL DENSITY
CFUNCTION   . Pr( Y<X)=PNORM(X)
CTHIS CODE IS FROM MONTOGOMERY,D.C.[1991,Page 447].
C
C=================================================I
FUNCTION PNORM(X)
C-------------------------------------------------
C Comment 12: Parameter TINY value is changable.
CWhen Y>10 then PNORM is considerably small.
C
C-------------------------------------------------
      PARAMETER(TINY1=1.E-60)
DOUBLE PRECISION C(7)
DOUBLE PRECISION Y,T,S
DATA C/0.319381530,-.356563782,1.781477937
  + ,-1.821255978,1.330274429D0,0.231641900D0,
```

```
   +  2.506628725D0/
IF(X.EQ.0.D0)THEN
   X=.00005D0
ENDIF
Y=X
IF(X.LT.0)Y=-X
T=1./(1.+C(6)*Y)
S=((((C(5)*T+C(4))*T+C(3))*T+C(2))*T+C(1))*T
IF(Y.GE.10.)THEN
    PNORM =TINY1
  ELSE
    PNORM=S*EXP(-Y*Y/2)/C(7)
ENDIF
IF(X.GT.0.)PNORM=1.-PNORM
RETURN
END

C=====================================================I
CSUB PROGRAM:10
CCUMULATIVE  BINOMIAL DISTRIBUTION FUNCTION
```

123

```
CTHIS SUBPROGRAM CALCULATE CUMULATIVE BINOMIAL
CDISTRIBUTION FUNCTION .HERE NU=SAMPLE NO.
CKU=ALLOWABLE NO. OF DEFECTIVES
CP=PROBABILITY OF DEFECTIVE  ITEM.
C
          FUNCTION  CUMB(NU,KU,P)
C----------------------------------------------------
C Comment 13: Like  Comment 12. TINY and TINY1
C  are changable
C
C----------------------------------------------------
PARAMETER(TINY=-205.0,TINY1=1.E-70)
KD=0
CUMB=0.0
Q=1.-P
IF (Q.EQ.0.D0)Q=TINY1
IF (P.EQ.0.D0)P=TINY1
NX=NU
KX=KU
   9 JU=NX-KD
BICON1=(FACTLN(NX)-FACTLN(KD)-FACTLN(JU)+KD*
```

```
     + ALOG(P)
     + +JU*ALOG(Q))
IF(BICON1.LE.-175)THEN
   T=0.
ELSE
   T=EXP(BICON1)
ENDIF
CUMB=CUMB+T
KD=KD+1
IF(KD.LE.KX)GO TO 9
RETURN
END
C==================================================I
CSUB PROGRAM:11
CFACTORIAL CALCULATION
CFOR LARGE VALUES  OF INTEGER VALUES, FACTORIAL
CFUNCTION WILL OVERFLOW ANYWAY. THAT IS WHY THIS
CSUBPROGRAM CALCULAT
CFLOATING POINT NO. OF Ln(N!) USING GAMMA FUNCTION
C.THIS CODE IS FROM Press et. all[1987]
```

```
C
FUNCTION FACTLN(LA)
C--------------------------------------------------
C Comment 14: Langth of dimension is changable
C  according to sample size. Change other values
C  accordingly.
C--------------------------------------------------
DIMENSION A(50)
DATA A/50*-1./
LX=LA
IF(LX.LT.0)PAUSE 'NEGATIVE FACTORIAL'
IF(LX.LE.49)THEN
    XU=LX+1.
   IF(A(LX+1).LT.0.) A(LX+1)=GAMMLN(XU)
   FACTLN=A(LX+1)
  ELSE
   FACTLN=GAMMLN(XU)
 ENDIF
 RETURN
 END
 C==================================================I
```

```
CSUB PROGRAM:12
CGAMMA FUNCTION CALCULATION
CTHIS CODE IS FROM Press et. all[1987]
C
FUNCTION   GAMMLN(XX)
REAL*8 COF(6),STP,HALF,ONE,FPF,X,TMP,SER
DATA COF,STP/76.18009173D0,-86.50532033D0,24.
     + 01409822D0,-1.231739516D0,.120858003D-2,
     + -.536382D-5,2.50662827465D0/
DATA HALF,ONE,FPF/0.5D0,1.0D0,5.5D0/
X=XX-ONE
TMP=X+FPF
TMP=(X+HALF)*LOG(TMP)-TMP
SER=ONE
DO 11 J=1,6
X=X+ONE
SER=SER+COF(J)/X
  11 CONTINUE
GAMMLN=TMP+LOG(STP*SER)
RETURN
END
```

```
C====================================================I
CSUB PROGRAM:13
CCUMULATIVE POISSION DISTRIBUTI  PART a:
C  THIS SUBROUTINE RETURNS INCOMPLETE GAMMA FUNCTION
CWHICH IS ALSO EQUAL TO CUMULATIVE POISION
CDISTRIBUTION FUNCTION. WHERE(A+1)=NUMBER OF
CACCEPTABLE UPPER LIMIT(Integer value) OF THE
CATTRIBUTE QUALITY CHARACTERISTIC.
CTHIS CODE IS FROM Press et. all[1987]
C
 FUNCTION GAMMQ(A,X)
 IF(X.LT.0.OR.A.LE.0.)PAUSE
 IF(X.LT.A+1.)THEN
     CALL GSER(GAMSER,A,X,GLN)
     GAMMQ=1.-GAMSER
   ELSE
     CALL GCF(GAMMCF,A,X,GLN)
     GAMMQ=GAMMCF
 ENDIF
 RETURN
```

```
 END
C====================================================I
CSUB PROGRAM:14
CCUMULATIVE POISSION DISTRIBUTION  PART b:
CTHIS SUBROUTINE IS A PART OF SUBPROGRAM: 13
CTHIS CODE IS FROM Press et. all[1987]
C
 SUBROUTINE  GSER(GAMSER,A,X,GLN)
 PARAMETER(ITMAX=100,EPS=3.E-7)
 GLN=GAMMLN(A)
 IF(X.LE.0.)THEN
    IF(X.LT.0.)PAUSE
    GAMSER=0.
    RETURN
 ENDIF
 AP=A
 SUM=1./A
 DEL=SUM
 DO 11 N=1,ITMAX
 AP=AP+1.
 DEL=DEL*X/AP
```

123

```
 SUM=SUM+DEL
 IF(ABS(DEL).LT.ABS(SUM)*EPS)GO TO 1
 11  CONTINUE
 PAUSE 'A TOO LARGE, ITNAX TOO SMALL, SUB:14'
 1  GAMSER=SUM*EXP(-X+A*LOG(X)-GLN)
 RETURN
 END
C==================================================I
CSUB PROGRAM:15
CCUMULATIVE POISSION DISTRIBUTION   PART c:
CTHIS SUBROUTINE IS A PART OF SUBPROGRAM: 13
CTHIS CODE IS FROM Press et. all[1987]
C
SUBROUTINE GCF(GAMMCF,A,X,GLN)
        PARAMETER(ITMAX=100,EPS=3.E-7)
        GLN=GAMMLN(A)
GOLD=0.
A0=1.
A1=X
B0=0.
```

```
B1=1.
FAC=1.
DO 11 N=1,ITMAX
AN=FLOAT(N)
ANA=AN-A
A0=(A1+A0*ANA)*FAC
B0=(B1+B0*ANA)*FAC
ANF=AN*FAC
A1=X*A0+ANF*A1
B1=X*B0+ANF*B1
IF(A1.NE.0.)THEN
     FAC=1./A1
     G=B1*FAC
     IF(ABS((G-GOLD)/G).LT.EPS)GO TO 1
     GOLD=G
ENDIF
  11 CONTINUE
PAUSE 'A TOO LARGE, ITMAX TOO SMALL,SUB:15'
   1 GAMMCF=EXP(-X+A*ALOG(X)-GLN)*G
RETURN
END
```

```
C=========================================================

CSUBPROGRAM :16
CTHIS CALCULATES NUMERATOR OF EXPECTED NO. OF DEFECTIVES
CIN A SAMPLE GIVEN THAT THE LOT IS REJECTED

C=========================================================

          FUNCTION RCLT(N,K,P,R)

           PARAMETER(TINY=1.E-20)

           NNX=n

           KD=k+1

           RCLT=0.0

           IF(P.EQ.0.DO)P=TINY

           Q=1.-P

           IF(P.EQ.0.DO)P=TINY

           IF(Q.EQ.0.DO)Q=TINY

    9      JU=NNX-KD

           BICON1=(FACTLN(NNX)-FACTLN(KD)-FACTLN(JU)+KD*ALOG(P)

     +      +JU*ALOG(Q)+ALOG(R))

            IF(BICON1.LE.-70)THEN

             BICON2=0.

             ELSE

              BICON2=KD*EXP(BICON1)
```

```
      END IF

      RCLT=RCLT+BICON2

      KD=KD+1

c     write(*,*)rclt,n,kd,p,r

      IF(KD.LE.Nnx)GO TO 9
      RETURN

      END

C==============END OF TARGETING PACKAGE==============I
```

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